Money in small communities

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Abstract

This paper investigates circulations of money in small communities. We build a two-player repeated gift-giving game and then show that players can sustain cooperation by using money. An efficient outcome is obtained when players are able to hold multiple units of currency.

Keywords: primitive money, repeated game.
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1 Introduction

This paper analyzes circulations of money in small communities. Since Kiyotaki and Wright (1989), the literature of monetary search theory has emphasized that money is essential under decentralized trade and lack of double coincidence of wants. Kocherlakota (1998) and Araujo (2004) show that existence of money also needs a lack of public information and contagion process of deviations. These conditions are satisfied when the number of members in a society is large. Hence theories of money as a medium of exchange usually assume large population economies.

On the other hand, there are some evidence that money was used in small communities. Some primitive monies were traded in local villages such as stone money, sea shells, crops, etc.[1] Douglas (1958) reports an example from anthropological fieldwork in Lele in Congo. Lele community was small and there was no market. The commercial trades were little. However, it is known that Lele people used a money. It is a cloth which is called Raffia. Douglas describes that this cloth currency was not mainly used as a medium of exchange but payments for social statuses, e.g., marriage due, age-set due, fine, fee to enter a cult, and so forth. The cloth currency had a role of keeping incentives of service to the community. In this sense, money was used as a medium of cooperation in this small society.

The present paper analyzes such a cooperative role of money in a two-player repeated gift-giving game. Then we show that money is a simple and efficient device for sustaining cooperation. The game is an example of a small society like Lele. A player receives a chance of gift-giving randomly and it is not observed by the other. Players have an incentive to hide the chance and do not give a gift. We show that, if there is one unit of fiat money, there exists an equilibrium that a player gives a gift when the opponent pays the money. It is a simple and intuitive strategy for small societies. In this equilibrium, the money is a device of record-keeping as noted in Kocherlakota (1998). Having the money means a player cooperated in a past period. Next we study a case that players are able to hold multiple units of money. Then it is shown that the first-best outcome is obtained when players' time preference converges to zero. In this sense, this simple strategy has some desired property about efficiency.

The game is classified as a repeated game with two-sided hidden states. This aspect is similar to Athey and Bagwell (2001) in a context of collusion between oligopoly firms. The analysis of holding multiple units of money is related to some search models such as Green and Zhou (1998), Zhou (1999), Camera and Corbae (1999) and Kamiya and Shimizu (2007).

[1] Goldberg (2005) criticizes that economists have considered some primitive currencies as fiat money incorrectly.

2 One unit of money

This section examines a simple case in which there is one unit of money. Let's consider a two-player repeated game in continuous time.[2] Each player gets a chance of giving one unit of gift stochastically. The arrival of the chance follows an independent Poisson process with parameter σ. The chance is private information so the opponent cannot observe. If a player gives a gift, it costs c and the opponent acquires a payoff u. Simply assume $u > c$. Each player discounts the payoff with time preference rate r.

One of the players has a unit of money. Now we consider a strategy that, if a player who does not have the money gives the gift, the owner of money pays it in exchange. Let V_1 denotes the expected discounted payoff of the money holder and V_0 of a player who does not have. On the equilibrium path, these satisfy

$$rV_0 = \sigma(-c + V_1 - V_0), \tag{1}$$
$$rV_1 = \sigma(u + V_0 - V_1). \tag{2}$$

The incentive constraint for gift-giving is

$$V_1 - V_0 - c \geq 0. \tag{3}$$

On the strategy, all past histories of the game is summarized only as money holding. Hence, the incentive constraint avoids the one-shot deviation condition so it is sufficient to hold an equilibrium. By (1) and (2), the condition (3) is rewritten as

$$u \geq \left(1 + \frac{r}{\sigma}\right) c. \tag{4}$$

The condition is satisfied when players are sufficiently patient. In this equilibrium, the ex-ante payoff is

$$\frac{r(V_0 + V_1)}{2} = \frac{\sigma(u - c)}{2}, \tag{5}$$

which is less than the efficient outcome $\sigma(u - c)$. This is because money holder does not give the gift when she gets a chance. In order to avoid this inefficiency, we allow players to hold multiple units of money in the next section. It decreases the probability that one of the two players holds no money.

3 Multiple units of money

On the same environment, now we suppose that the sum of money holdings is N units. Let V_i denotes the expected discounted payoff when a player have i units of money. Let's consider the same strategy that a player pays one unit of money if she receives a gift. The equilibrium values satisfy

$$rV_0 = \sigma(-c + V_1 - V_0) \tag{6}$$
$$rV_n = \sigma(-c + V_{n+1} - V_n) + \sigma(u + V_{n-1} - V_n) \quad \text{for } n = 1, \dots, N-1 \tag{7}$$
$$rV_N = \sigma(u + V_{N-1} - V_N) \tag{8}$$

and the conditions for the equilibrium become

$$\forall i \in \{0, 1, \dots, N-1\}, \quad V_{i+1} - V_i - c \geq 0. \tag{9}$$

Proposition 1. *For arbitrary $N \in \mathbb{N}$, there exists $r_N > 0$ such that the equilibrium exists for all r where $0 < r \leq r_N$.*

On this strategy, a player gives a gift and receives a unit of money in order to avoid a case in which her money holding runs out. This is a risk that the player cannot receive a gift even if the other gets a chance. When N is large, the probability of such cases becomes low so a welfare will be high. On the other hand, since the risk is low, the incentive constraints may violate. This is because money will be exhausted only far future. In order to sustain the incentive of gift-giving, the time preference r needs to be low. This proposition states that in the limit case $r \to 0$, the incentive constraints hold for all N. In a case $N \to \infty$, the inefficiency from holding no money is eliminated; hence the first best outcome is obtained.

[2] We do not use a standard discrete time repeated game just for simplification.

Proof. First we solve the difference equation of V_n. By (7),

$$\sigma V_{n+2} - (r + 3\sigma)V_{n+1} + (r + 3\sigma)V_n - \sigma V_{n-1} = 0,$$

which is a third order difference equation. The characteristic equation is

$$\sigma\alpha^3 - (r + 3\sigma)\alpha^2 + (r + 3\sigma)\alpha - \sigma = (\alpha - 1)(\sigma\alpha^2 - (r + 2\sigma)\alpha + \sigma) = 0. \tag{10}$$

Let me define $k \equiv r/\sigma$. The solutions of (10) are

$$1, \quad 1 + \frac{k}{2} + \sqrt{\frac{k^2}{4} + k} \text{ and } 1 + \frac{k}{2} - \sqrt{\frac{k^2}{4} + k}.$$

Let A and B denotes the second and third solutions. Then $A > 1$, $0 < B < 1$, $A + B > 1$. Let C_1, C_2, C_3 be constant coefficients then the general solution of (10) can be written as

$$V_n = C_1 + C_2 A^n + C_3 B^n. \tag{11}$$

By (7),

$$\sigma(C_2 A^{n+2} + C_3 B^{n+2}) - (r + 2\sigma)(C_2 A^{n+1} + C_3 B^{n+1}) + \sigma(C_2 A^n + C_3 B^n) = rC_1 - \sigma(u - c). \tag{12}$$

Since A and B are solution of (10) except 1, the left hand side of (12) is 0. Then

$$C_1 = \frac{\sigma}{r}(u - c).$$

By substituting (11) into (6) and (8),

$$C_2[(1 + k)A^N - A^{N-1}] - C_3[B^{N-1} - (1 + k)B^N] = c \tag{13}$$

$$C_2[A - (1 + k)] - C_3[(1 + k) - B] = u \tag{14}$$

Let me define $D \equiv (1 + k)A^N - A^{N-1}$, $E \equiv B^{N-1} - (1 + k)B^N$, $F \equiv A - (1 + k)$ and $G \equiv (1 + k) - B$. It can be easily shown that $D > 0$, $E > 0$, $F > 0$, $G > 0$. By (13) and (14),

$$C_2 = \frac{Eu - Gc}{EF - DG}, \quad C_3 = \frac{Du - Fc}{EF - DG}. \tag{15}$$

It is always holds $EF - DG < 0$ and $D > F$. We will show $G/E \to 1$ as $r \to 0$ later so temporary we assume it. If r is sufficiently close to 0, $C_2 < 0$ and $C_3 < 0$. By (11),

$$\begin{aligned} c &= C_2[(1 + k)A^N - A^{N-1}] - C_3[B^{N-1} - (1 + k)B^N] \\ &< C_2[(1 + k)A^n - A^{n-1}] - C_3[B^{n-1} - (1 + k)B^n] \\ &< C_2[A^n - A^{n-1}] - C_3[B^{n-1} - B^n] = V_n - V_{n-1} \end{aligned}$$

satisfy for all $n \leq N - 1$. The incentive constraint (9) is satisfied.

Finally, we show $G/E \to 1$ as $r \to 0$. Obviously $k \to 0$ as $r \to 0$.

$$\frac{G}{E} = \left[\frac{1}{\left(1 + k/2 - \sqrt{(k^2/4) + k}\right)^{N-1}}\right] \cdot \left[\frac{(1 + k) - \left(1 + k/2 - \sqrt{(k^2/4) + k}\right)}{1 - (1 + k)\left(1 + k/2 - \sqrt{(k^2/4) + k}\right)}\right]$$

The first bracket converges to 1 and, by applying L'Hopital's rule, it can be shown that the second bracket also converges to 1. By $1 + k/2 - \sqrt{(k^2/4) + k} = (1 + k/2) - \sqrt{(1 + k/2)^2 - 1} < 1$, in order to put G/E close to 1, r must be nearer to 0 as N becomes larger. □

4 Conclusion

This paper shows that a simple strategy using money sustains cooperation on a two-player repeated gift-giving game. We also show that, when time preference converges to 0, the efficient equilibrium can be obtained. This result justifies the circulations of primitive money in small communities.

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