Abstract

We develop a closed economy model in order to study the interactions among sovereign risk premia, fiscal limits and fiscal policy. Default risk premia reflect the market’s expectations about the ability and willingness of the government to service its debt. The government’s ability arises endogenously from fiscal limits implied by Laffer curves, while the government’s willingness is determined by the flexibility of its fiscal policy. The model rationalizes different sovereign ratings across developed countries. The distribution of fiscal limits is country specific and depends on the size of the government, the degree of the countercyclical policy responses, economic diversity and political uncertainty. The model also produces a nonlinear relationship between sovereign risk premia and the level of government debt. In recessions, the default risk premia of long-term bonds jump ahead of short-term bonds and provide early warnings of sovereign defaults. The nonlinearity is consistent with the empirical evidence that once risk premia begin to rise, they do so rapidly. In addition, the model predicts substantial risk premia that are close to those observed, even when the probability of defaulting is remote. Risk premia carry substantial economic costs, lowering consumption and output for extended periods.

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1 Introduction

As a result of the deterioration of public finances in tandem with mounting economic pressure, rating agencies downgraded the sovereign debt in several OECD countries in the beginning and middle of 2009. Greece, Portugal, and Spain were downgraded by one notch in January, while Ireland was downgraded by one notch in March and a second notch in June\(^1\) The spread between 10-year Irish government bonds and equivalent German bonds widened to approximately 250 basis points in June of the same year. Even the United Kingdom had its sovereign rating revised to a negative watch in May. Contrary to conventional wisdom, developed countries are frequently penalized when the financial market raises concerns about the risk of government debts. The large build-up of public debt across the OECD countries in recent decades has produced a series of downgrades in government debts. Figure\(^2\) depicts the sovereign ratings since 1975.\(^3\) This evidence indicates that the joint analysis of sovereign default risk premia and fiscal policy behavior is an important line of theoretical work.

This paper studies the interactions among sovereign default risk premia, fiscal limits and fiscal policy in a general equilibrium framework in which Laffer Curves, arising from distortionary taxation, constrain the government’s ability to service its debt.\(^4\) Default risk premia reflect the market’s expectations about the ability and willingness of the government to fulfill its bond contracts. The model can rationalize the sovereign ratings and resulting risk premia across developed countries and it also predicts that the default risk premium rises nonlinearly with respect to the level of the government’s debt. The default risk premia of long-term bonds jump ahead of short-term bonds and provide early warnings of sovereign defaults. Furthermore, even under a remote probability of sovereign defaulting, the model predicts substantial default risk premia that can lower private consumption and aggregate output over prolonged periods.

In the first part of the paper, we develop a two-period model that admits analytical solutions and highlights the mechanism through which the Laffer Curves affect sovereign default risk. The government may have to renege on its debt if it reaches the peak point of the Laffer Curve and is unable to raise more tax revenue by levying higher tax rates. We find that the government’s ability to service its debt varies with the underlying macroeconomic fundamentals. A government with a heavy burden of fiscal transfers has a high probability of reaching the peak point of the Laffer Curve and, therefore, may face a high borrowing cost. Strong countercyclical transfers, which arise from large automatic stabilizers or discretionary countercyclical fiscal policies, deteriorate government budgets in recessions and increase sovereign default risks. A diversified economic structure that is less vulnerable to exogenous shocks may

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\(^1\) All sovereign ratings are from Standard & Poor’s.
\(^2\) France, Germany, Norway and the United States have not been downgraded by Standard & Poor’s since their sovereign ratings became available.
allow the government to raise tax revenue without entering the the slippery side of the Laffer Curve. In addition, the probability of the government to fulfill its bond contract also hinges on political uncertainty. These findings are consistent with the sovereign ratings across developed countries, which are discussed below.

In the second part of the paper, we further explore the interactions between sovereign risk premia and fiscal limits by solving an infinite-horizon model numerically. In this model, a stochastic fiscal limit, which measures the maximum level of debt that the government is able to service, endogenously arises from the Laffer Curves. The distribution of the stochastic fiscal limit depends on the underlying macroeconomic fundamentals through a similar channel as discussed in the two-period model. We consider a closed economy in which the government finances an exogenous level of purchases and countercyclical lump-sum transfers to households by distorting taxation and bond issuance. However, the bond contract is not enforceable. The government may renege on a fixed fraction of its debt, if the level of government debt surpasses the stochastic fiscal limit. Households are assumed to know the distribution of the fiscal limit. Using this information, they can decide the quantity of government bonds that they are willing to hold and the price at which they are willing to purchase the bonds. We find that the default risk premium, which is proportional to the probability of sovereign default, is a nonlinear function of the level of the government’s debt. The risk premium begins to emerge as the level of the debt approaches the fiscal limit and it can lower private consumption and aggregate output over prolonged periods. In addition, we find that the default risk premia of the long-term bonds jump ahead of the short-term bonds and provide early warnings of sovereign defaults.

We then use the infinite-horizon model to discuss the impact of fiscal policies on the economy. The flexibility of fiscal policy reflects the willingness of the government to service its debt. If the government is flexible in that it can raise the tax rate or cut government expenditures (or both), then it can steer through recessions without jeopardizing its sovereign rating. First, we consider economies where governments differ in their willingness to retire sovereign debt by raising tax rates. Second, government purchases are no longer exogenous; instead, they are modeled as policy instruments that follow a regime-switching rule.

Finally, we present an extension of the infinite-horizon model. The portion of debt that the government default is no longer fixed; instead, it depends upon the state of the economy. The variation does not change the main results presented in the basic model.

Before launching into the models in Section 2 and 3, we first discuss different sovereign ratings across the developed countries and then present the related literature.

**Sovereign ratings:** The predictions of the model are consistent with the sovereign ratings across the OECD countries.
First, large fiscal transfer programs generate small fiscal margins for maneuver and place a short leash on the level of government debt. For instance, Figure 2 shows that since 1980, the United Kingdom has always had lower levels of net debt as a share of the GDP than the United States. In addition, government gross debts are expected to climb to 90% of the GDP by 2013 in both countries. While Standard & Poor’s revised the outlook of the sovereign rating of the United Kingdom to a negative watch in 2009, it claimed that “it is unlikely that the ratings on the United States government will be lowered in the near term”. One explanation for such a discrepancy is that the United Kingdom has a heavier burden in regard to fiscal transfers and, therefore, a higher level of tax pressure. The average tax rate in the United Kingdom has consistently exceeded the rate in the United States by nearly 10% since 1980.

Second, a diversified economy that is less vulnerable to shocks has a greater capacity in regard to economic adjustment. For example, when comparing Canada and New Zealand, both countries have similar average tax rates (32.4% in Canada and 30.9% in New Zealand) and comparable average government spending shares (22.9% of the GDP in Canada and 20.8% of the GDP in New Zealand). However, the two countries have been treated differently by the rating agencies, as is illustrated in Figure 3. From 1983 to 1992, New Zealand government’s debt rating was reduced by three notches from AAA to AA-, as its net debt climbed from 30% to 50% of the GDP. The Canadian government, on the other hand, was able to keep its AAA rating until its net debt hit 60% of the GDP in 1992. The rating was reduced by one notch to AA+ and then stayed at this level until 2001. An important reason for this discrepancy is that the economy of New Zealand is less diversified than the economy of Canada. For instance, Standard & Poor’s sees New Zealand economy’s structure as relatively narrowly-based and heavily reliant on agriculture. This economic structure makes New Zealand particularly vulnerable to international commodity price fluctuations and global economic slowdowns.

Third, an inadequate policy response or a lack of political will to act on the deterioration of public finances raise political uncertainty and are often important factors in sovereign downgrades. Such downgrades can be seen in a comparison of Italy and Belgium. Both Italy and Belgium accumulated massive public debt in the 1990s, well above 100% of the GDP, as shown in Figure 4. However, only the Italian government’s debt received persistent downgrades, while Belgium’s rating has been stable. One possible explanation is that Belgium made great strides in reducing its total government spending and net transfers, by 10% of the GDP since 1980 as can be seen in the top right panel of Figure 5. The resulting reduction in the fiscal deficit,

\[4\] The projection of the U.S. debt is from Standard & Poor’s Rating Direct (2009). The projection for the U.K. is from the HM Treasury Budget (2009).
\[5\] The average tax rate is defined as the ratio of the total tax revenue over GDP.
\[6\] Standard & Poor’s (2007) reported that agriculture accounted for close to 7% of New Zealand’s GDP and 58% of its export receipts. In fact, dairy alone accounted for 20% of its export receipts.
\[7\] Total government spending include government consumption of fixed capital and government final consumption of expenditures. Net transfers are the sum of social security payment, net capital transfer and subsidies.
from 12.7% of the GDP in 1981 to 5.5% in 1991, demonstrated strong political willingness to be fiscally responsible. On the contrary, a high level of public debt has been sustained in Italy since 1980 in spite of fiscal consolidation attempts that have occurred periodically in the country since the early 1990s. The top left panel of Figure 5 shows that total government spending and net transfers have increased by 10% of the GDP since 1980.

**Related literature:** This paper relates to a growing empirical literature that identifies the nonlinearity of sovereign default risk premia. Bayoumi, Goldstein and Woglom (1995) found a strong nonlinear relationship between municipal bond yields and debt variables for U.S. states. Alesina, De Broeck, Prati and Tabellini (1992) compared 12 OECD countries and found that sovereign default risks were affected by the debt level at high levels of debt, but not influenced by the debt level at low levels of debt. Ardagna, Caselli and Lane (2007) also identified a nonlinear effect of government debt on interest rates using a panel data of 16 OECD countries over several decades. Bernoth, von Hagen and Schuknecht (2006) focused on European countries between 1993 and 2005 and found that the debt service ratio raised the spreads nonlinearly. A recent study from the OECD by Haugh, Ollivaud and Turner (2009) analyzed large movements in the sovereign yield spreads between Germany and other European countries in the current financial crisis and found that deteriorations in fiscal performance increased the spread in a nonlinear way.

The explanation of sovereign default has been debated in this field since 1980s. In Kharas’ (1984) study, he focused on the government’s ability to service its debt, assuming that both the tax policy and saving rate were exogenous. Uribe (2006) considered sovereign default that was determined by the transversality condition due to an exogenous tax policy. On the other hand, Eaton and Gersovitz (1981), and Eaton, Gersovitz and Stiglitz (1986) emphasized the willingness of the government to service its debt. They argued that models which view critical parameters of the economy as exogenous may be harmful to policymaking. The growing literature on international borrowing in emerging markets has followed the approach of Eaton and Gersovitz (1981) (see Arellano (2008), Aguiar and Gopinath (2006), Yue (2006) among many others). The literature predicts that governments may default at levels of debt that are much lower than those observed. In addition, it may be more realistic to assume that the government cannot completely ignore powerful domestic stakeholders and simply default at will on domestic debt, especially in developed countries. We contribute to the debate by jointly considering a country’s ability and willingness to pay. The model can rationalize the sovereign ratings and resulting risk premia across the OECD countries.

8Arellano (2008) obtained a debt ratio of 6% of the GDP on average, Yue (2006) obtained a ratio of 9.7% of the GDP, and Aguiar and Gopinath (2006) obtained a ratio of 27% of the GDP by mismatching the default probability with the data. Mendoza and Yue (2008) generated a debt ratio of 26% of the GDP on average and a default probability of 3%. A crucial assumption is that domestic producers of final goods have to obtain working capital loans from abroad for imported intermediate goods.
In fiscal policy research, an overwhelming majority of the theoretical models assume that sovereign debts are always honored\footnote{Distinguished exceptions include Sargent and Wallace (1981), Uribe (2006), Cochrane (2009) and Davig, Leeper and Walker (2009).} These models include those where the timing of the taxation is irrelevant due to Ricardian equivalence (see Barro (1976)), where fiscal policy is a crucial player in determining the equilibrium (see Leeper (1991) and Woodford (1995) among many others), and where the optimal path of debt is a random walk due to tax smoothing (See Barro (1979), Lucas and Stokey (1983), and Aiyagari, Marcet, Sargent and Seppala (2002)). It is at odds with the fact that countries with rampant fiscal policies are frequently penalized by the financial market. Allowing sovereign default in these models can potentially be very important. For example, Pouzo (2009) extends the model presented by Aiyagari, Marcet, Sargent and Seppala (2002) in order to show that the optimal level of debt ceases to be a random walk once the government has the option of default. We propose a framework with endogenous fiscal limits that can account for sovereign ratings across developed countries and is flexible enough to be adopted in a broader analysis.

2 Two-period Model: Analytical Intuition

Before launching into the numerical analysis of an infinite-horizon model, we examine a two-period model that admits analytical solutions and highlight the mechanism through which Laffer Curves affect sovereign risk premia.

We consider a closed economy with exogenous and stochastic productivity in each period. The productivity ($A$) follows a Markov process with $A \in \{A_L, A_H\}$ and transition matrix $P$ specified as,

$$
\begin{bmatrix}
P_{LL} & P_{LH} \\
P_{HL} & P_{HH}
\end{bmatrix} =
\begin{bmatrix}
p & 1-p \\
1-p & p
\end{bmatrix}.
$$

Technology is linear and the total amount of time is normalized to one.

The government: The government finances a fixed amount of lump-sum transfers to household by levying tax on labor income and issuing one-period bond. Without loss of generality, we assume that the government purchases are zero and the initial government debt is also nil. The amount of transfers, therefore, represents the size of the government.

In period 1, we assume that the economy receives low productivity and that the government issues one-period bond ($b_1$), instead of increasing tax rate, to finance the fixed amount of transfers ($\bar{S}$). $q_1$ is the bond price in terms of consumption and $1 - L_1$ is the labor supply.

$$
\bar{S} = q_1 b_1 + \bar{\tau} A_1 (1 - L_1)
$$

\footnote{Distinguished exceptions include Sargent and Wallace (1981), Uribe (2006), Cochrane (2009) and Davig, Leeper and Walker (2009).}
In period 2, the government raises the tax rate in order to pay back its debt. However, a tax rate exists, for a given state of the economy $A_i$ ($i = \{L, H\}$), such that higher rates do not raise more revenue. This point is the peak point of the Laffer Curve and is denoted as $\tau_i^{\text{max}}$. The Laffer Curves constrain the maximum level of tax revenue that the government can collect in period 2, depending on the state of the economy. The government may have to renege on its debt if the tax revenue falls short. The portion that it may default ($\Delta_2$) is determined by its budget constraint in period 2.

$$\bar{S} + (1 - \Delta_2)b_1 = \tau_2A_2(1 - L_2)$$ (2)

The household: With access to the sovereign bond market, the household chooses consumption ($c$) and leisure ($L$) to maximize its welfare.

$$\max E_1(u(c_1, L_1) + \beta u(c_2, L_2))$$ (3)

$$\text{s.t. } A_1(1 - \tau_1)(1 - L_1) + \bar{S} - c_1 = b_1q_1$$ (4)

$$A_2(1 - \tau_2)(1 - L_2) + \bar{S} - c_2 = -(1 - \Delta_2)b_1$$ (5)

The household’s first-order conditions require that the marginal rate of substitution between consumption and leisure satisfies,

$$\frac{u_L(t)}{u_c(t)} = A_t(1 - \tau_t) \quad (t = 1, 2)$$ (6)

and the price of the government debt satisfies,

$$q_1 = \beta E_1 \left( (1 - \Delta_2) \frac{u_c(c_2)}{u_c(c_1)} \right)$$ (7)

Equation (7) shows that the bond price in period 1 reflects the household’s expectation about sovereign default in period 2. The higher the default portion ($\Delta_2$), the lower the bond price.

Laffer Curve and default: We assume the utility function to be quasi-linear.

$$u(c, L) = c + \sigma \log L$$ (8)

The unitary marginal utility of consumption implies that the labor supply only depends on the level of productivity and the tax rate. For each level of productivity ($A_i$), there exists a maximum level of tax revenue ($T_i^{\text{max}}$) that the government is able to collect. Appendix A shows the derivation.

$$T_i^{\text{max}} = \sqrt{A_i} \left( \sqrt{A_i} - \sqrt{\sigma} \right)^2 \quad i = \{L, H\}$$ (9)

$$\tau_i^{\text{max}} = 1 - \sqrt{\frac{\sigma}{A_i}}$$ (10)
In period 2, the government may need to default if the tax revenue falls short of the payments of debt and transfers. In order to derive a closed-form solution, $A_L$ is assumed to be sufficiently low so that the maximum level of tax revenue at the low level of productivity can barely cover the payment of transfers. Therefore, the government has to default if the level of productivity is low in period 2. On the other hand, $A_H$ is assumed to be sufficiently high so that the government does not need to default if the level of productivity is high in period 2. The default rate ($\Delta_2$) can be summarized as,

$$\Delta_2 = \begin{cases} 
0 & \text{if } A_2 = A_H \\
> 0 & \text{if } A_2 = A_L 
\end{cases}$$

**The solution:** The bond price in period $t = 1$ can be solved in terms of the level of transfers ($\bar{S}$), the probability of staying in the recession in period 2 ($p$), the maximum level of tax revenue at the low level of productivity ($T_{L}^{\max}$) and the tax revenue collected in period 1 ($T_1$). Appendix A shows the derivation.

$$q_1 = \frac{\beta(1 - p)}{1 - \beta p \frac{T_{L}^{\max} - \bar{S}}{S - T_1}}$$

where $T_{L}^{\max}$ is defined in Equation (9) and $T_1$ is,

$$T_1 = \bar{\tau} (A_L (1 - \bar{\tau}) - \sigma)$$

In addition, the quasi-linear utility function implies that the risk-free bond price is constant,$q_{f1} = \beta$. The default risk premium is,

$$r_1^{\Delta} = \frac{1}{q_t} - \frac{1}{q_{f1}} = \frac{p}{\beta(1 - p)} \left( 1 - \beta \frac{T_{L}^{\max} - \bar{S}}{S - T_1} \right).$$

### 2.1 Determinants of Default Risk Premia

**Government size:** Absent from government purchases, the size of the government is measured by the amount of lump-sum transfers. Equation (13) shows that the larger the size of the government, the higher the default risk premium. More transfers raise the fiscal deficit and the government has to default a larger portion of its debt if the level of productivity stays low in period 2. The household, therefore, is less willing to hold government bond, which raises the sovereign default risk premium.

**Shock persistence:** Equation (13) shows that a more persistent shock raises the default risk premium. A long-lasting recession lowers the tax revenue and raises the portion of the debt that government may have to default in a prolonged recession.
**Countercyclical transfers:** We revise the specification of the lump-sum transfers to be countercyclical in order to capture fiscal policy in many OECD countries. In practice, governments tend to transfer more resources to households in economic downturns through both automatic stabilizers and countercyclical discretionary fiscal policy. The parameter $\zeta$ measures the degree of the countercyclical transfers.

$$S_t - \bar{S} = -\zeta (A_t - \bar{A}) \quad (\zeta > 0) \quad \text{(14)}$$

The bond price can be rewritten as,

$$q_1 = \frac{\beta (1 - p)}{1 - \beta p \frac{T_{L,\text{max}} - S}{S - T_1}} = \frac{\beta (1 - p)}{1 - \beta p \frac{\bar{S} - \zeta (A - A_L)}{\bar{S} + \zeta (A - A_L) - T_1}} \quad \text{(15)}$$

A larger $\zeta$ lowers the bond price. In a recession, the government transfers more resources to households through generous unemployment benefits and other welfare programs. These transfers further deteriorate the government budget and raise the sovereign default risk premium.

**Political uncertainty:** So far, we have assumed that the government always pays back its debt unless it hits the peak of the Laffer Curve. In practice, governments might be constrained by some political limit that is much lower than the peak of the Laffer Curve.

We revise the specification of the model in period 2 and allow the following possibility: even if the level of productivity is high in period 2, the government may choose not to raise the tax rate but, instead, to renege on its debt. The default scheme can be summarized as following. If the productivity is low in period 2, the government hits the peak of the Laffer Curve and has to renege on its debt. If the productivity is high, with probability of $\theta$, the government may raise the tax rate and pay back its debt; with probability of $1 - \theta$, it may keep the tax rate constant at $\bar{\tau}$ and default part of its debt. The parameter $\theta$ measures political uncertainty. Appendix A shows that the interest rate of government bond can be written as,

$$R_1 \equiv 1 - \frac{T_2^H - \bar{S}}{S - T_1} + \frac{1 - \beta p \frac{T_{L,\text{max}} - \bar{S}}{\bar{S} - T_1} - \beta (1 - p) T_2^H S}{\beta \theta (1 - p)} \quad \text{(16)}$$

where $T_{L,\text{max}}$ is defined in Equation (9) and $T_1$ in Equation (12). $T_2^H$ is the tax revenue in period 2 if the tax rate stays at $\bar{\tau}$ and the level of productivity is high,

$$T_2^H = \bar{\tau} (A_H (1 - \bar{\tau}) - \sigma) \quad \frac{1}{1 - \bar{\tau}} \quad \text{(17)}$$

Equation (16) shows that a government with lower political uncertainty (higher $\theta$) may face a lower risk premium. Fiscal transparency, such as credible budgetary rules, may lead the private sector to believe that the government is fiscal responsible and therefore reduce the sovereign borrowing cost.
3 Infinite-horizon Model: Numerical Simulation

The two-period model in Section 2 highlights the mechanism through which the Laffer Curves constrain the government’s ability to service its debt. This mechanism remains at work in an infinite-horizon model where a stochastic fiscal limit, which measures the maximum level of debt the government is able to service, arises endogenously from the Laffer Curves. In this section, we solve the infinite-horizon model numerically in order to study the interactions among sovereign risk premia, fiscal limits and fiscal policy.

The model represents a closed economy with a government and a representative competitive household. Let $c_t$, $L_t$, $g_t$, and $A_t$ denote household consumption, leisure, government purchases, and productivity at time $t$. Technology is linear and the total amount of time is normalized to one. The aggregate resource constraint is

$$c_t + g_t = A_t (1 - L_t)$$  \hspace{1cm} (18)

Both productivity ($A_t$) and government purchases ($g_t$) follow AR(1) processes \(^{10}\)

$$\ln \frac{A_t}{A} = \rho^u \ln \frac{A_{t-1}}{A} + u_t \quad u_t \sim N(0, \sigma_u^2)$$  \hspace{1cm} (19)

$$\ln \frac{g_t}{g} = \rho^e \ln \frac{g_{t-1}}{g} + e_t \quad e_t \sim N(0, \sigma_e^2)$$  \hspace{1cm} (20)

The government: The government finances lump-sum transfers to households ($S_t$) and exogenous and unproductive purchases ($g_t$) by raising tax revenues and issuing one-period bonds ($b_t$). Tax revenue is raised through a time-varying flat rate tax ($\tau_t$) on labor income.

$$\tau_t A_t (1 - L_t) + b_t q_t = \underbrace{(1 - \Delta_t) b_{t-1}}_{b_t'} + g_t + S_t$$  \hspace{1cm} (21)

$q_t$ is the price of the bond in units of time $t$ consumption. For each unit of bond, the government promises to pay household one unit of consumption next period. However, the bond contract is not enforceable. At time $t$, the government may partially default on its liability ($b_{t-1}$) by the rate of $\Delta_t$. The post-default government liability is denoted as $b_t'$. Default is given by,

$$\Delta_t = \begin{cases} 
0 & \text{if } b_{t-1} < b_t' \\
\delta & \text{if } b_{t-1} \geq b_t'
\end{cases}$$

The default scheme depends on a stochastic fiscal limit ($b_t'$), which arises endogenously from Laffer Curves. Section 3.1 will provide a further discussion of $b_t'$.

\(^{10}\)In the section of extensions, $g_t$ will no longer be an exogenous process; instead, it will endogenously adjust to the level of government indebtedness.
We assume that the government follows a simple tax rule, which is an abstraction designed to capture the tax policy in practice. Across countries, fiscal authority tends to increase the tax rate when the level of public debt rises.

\[ \tau_t - \tau = \gamma \left( b_t^d - b \right) \]  

(22)

\( b_d^t \) is defined as \((1 - \Delta_t) b_{t-1} \). We refer to \( \gamma \) as the “tax adjustment parameter” \((\gamma > 0)\). A larger \( \gamma \) means that the government is more willing to finance debt by raising the tax rate. In addition, time-varying lump-sum transfers are modeled to capture countercyclical fiscal policy. Equation (23) posits that transfers increase in recessions. The parameter of \( \zeta \) is defined as the elasticity of transfers with respect to productivity.

\[ \ln \frac{S_t}{S} = -\zeta \ln \frac{A_t}{A} \]  

(23)

**The household:** The representative household chooses consumption, leisure, and bond purchases according to,

\[
\max E_t \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) 
\]

s.t. \( A_t (1 - \tau_t)(1 - L_t) + S_t - c_t = b_t q_t - (1 - \Delta_t) b_{t-1} \)  

(25)

with prices and policies, \( \{ \tau_t, S_t, q_t, \Delta_t \} \), taken as given. \( E_t \) is the mathematical expectation that is conditional on time \( t \) information, including the sovereign default information at time \( t \). \( \beta \) is the discount factor. \( u(c, L) \) is strictly increasing in household consumption and leisure. The household’s first-order conditions are,

\[
\frac{u_L(t)}{u_c(t)} = A_t (1 - \tau_t) 
\]

(26)

\[ q_t = \beta E_t \left( (1 - \Delta_t+1) \frac{u_c(t + 1)}{u_c(t)} \right) \]  

(27)

Equation (27) shows that the government bond price reflects the household’s expectation about sovereign default in the next period \((\Delta_t+1)\). The optimal solution to the household’s maximization problem must also satisfy the following transversality condition:

\[
\lim_{j \to \infty} E_t \underbrace{q_t q_{t+1} \cdots q_{t+j}}_{Q_{t, t+j}} b_{t+j} = 0 
\]

(28)

It can be rewritten as,

\[
\lim_{j \to \infty} \beta^{j+1} E_t \frac{u_c(t + j + 1)}{u_c(t)} \left( (1 - \Delta_{t+j+1}) b_{t+j} \right) = 0 
\]

(29)

Equation (29) is obtained by substituting the household first order condition, Equation (27), into the transversality condition, Equation (28). Appendix B shows the derivation.
3.1 Discussion of Fiscal Limit

The Laffer Curves remain at work in the infinite-horizon model: a tax rate ($\tau_{\text{max}}$) exists, for each state of the economy ($A, g$), such that higher rates do not raise more revenue. If it taxes labor income at $\tau_{\text{max}}$, the government can collect the maximum level of tax revenue and, therefore, run the maximum level of fiscal surplus for the given state of the economy. The ceiling of the fiscal surplus should constrain the maximum level of debt that the government will be able to pay back, denoted as $b^*_t$. Mathematically, $b^*_t$ is the expected sum of the discounted fiscal surplus if the government collects the maximum level of tax revenue from time $t$ on. $b^*_t$ can be written as,

$$b^*_t = E_t \sum_{h=0}^{\infty} (\beta \theta)^h \frac{u^\text{max}_c (t + h)}{u^\text{max}_c (t)} (T^\text{max}_{t+h} - g_{t+h} - S_{t+h})$$

(30)

where $\theta$ measures the political uncertainty ($\theta \leq 1$). $u^\text{max}_c$ and $T^\text{max}$ represent the marginal utility of consumption and the total tax revenue respectively, given that the tax rate is at $\tau_{\text{max}}$.

In this model, household consumption and labor supply only depend on the income tax rate and the exogenous state variables. Since the Laffer Curve has a unique peak point at each state of the economy, the distribution of the fiscal limit can be obtained using Markov Chain Monte Carlo simulation. We approximate the distribution as $\mathcal{N}(b^*, \sigma^2_t)$. Appendix C explains the numerical procedure.

**Determinants of fiscal limit:** Equation (30) shows that the fiscal limit is determined by the path of lump-sum transfers, the path of the tax revenue and the political uncertainty. The first two factors, in turn, depend upon the deep parameters of the model and shock processes. Figure (6) compares the distributions of fiscal limits as different parameters change.

**Political uncertainty:** If political parties with different policy goals alternate in power, or self-interested politicians are less patient than households, the government may consider a relatively short horizon when making policy decisions and, therefore, raise the political uncertainty. In Equation (30), future fiscal surplus is discounted not only by a pure rate of time preference ($\beta$), but also by an additional political factor ($\theta$). $\theta$ is a shortcut of political uncertainty which we have discussed in the two-period model in Section 2. This shortcut follows Grossman and Huyck (1988), and Cuadra and Sapriza (2008). Hatchondo, Martinez, and Sapriza (2007) summarized the literature that discusses the links between political risk and

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\(^{11}\)This statement is contingent on the assumption that no private capital or capital tax exist. In an economy with capital, Laffer Curves still exist and, therefore, a fiscal limit also exists. However, it could be more complex to find the fiscal limit.

\(^{12}\)Grossman and Huyck (1988) showed that there existed an equivalent representation between the probability of the current sovereignty survival and the prospective longevity of sovereign’s survival in power.
sovereign defaults. In addition, this setup is similar in spirit to how the rating agencies evaluate sovereign risks. Standard & Poor’s (2008) stated that “the stability, predictability, and transparency of a country’s political institutions are important considerations in analyzing the parameters for economic policymaking...”

The middle left panel of Figure (6) shows that a government with higher political uncertainty (a lower $\theta$) may face a tighter fiscal limit than an otherwise identical economy. Baldacci, Gupta and Mati (2008) provided empirical evidence that both fiscal and political factors mattered for credit risks in emerging markets.

**Countercyclical transfers:** Strong countercyclical fiscal transfers, arising from a large automatic stabilizer or discretionary countercyclical policy, can aggravate the volatility of the fiscal limit. In recessions, the government transfers more resources to households and deteriorate its budget. The bottom left panel of Figure (6) compares four economies with different degrees of countercyclical transfers. The economy with the largest $\zeta$ faces the most dispersed distribution of the fiscal limit.

**Government size:** The top right panel of Figure (6) considers four economies that are identical except for the size of governments, which are measured by the size of transfers in the steady state. The economy with a larger government faces a lower fiscal limit on average. There are two reasons. First, for a given level of tax revenue and government purchases, the larger the transfer program, the lower the fiscal surplus. Second, a larger size of government implies a higher tax rate at the steady state. The economy is closer to the slippery side of the Laffer Curve, which leaves less fiscal space to retire the government debt by further raising the tax rate in a recession.

**Shock processes:** Figure (6) also compares economies with different shock processes. The middle and bottom right panels show that the economy may face a dispersed distribution of fiscal limit if exogenous shocks are more persistent or more volatile. This comparison implies that New Zealand, an economy that is more vulnerable to external shocks, may have lower boundaries in regard to its fiscal limit than a more diversified economy such as Canada. More broadly, it is consistent with empirical evidence that differences in underlying macroeconomic volatility are important to explain why many emerging market economies with moderate debt burdens face higher spreads than other countries with far higher debt ratios (see Catao and Kapur (2006)).
3.2 Method and Calibration

Method: The solution method, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the system to a set of expectation first-order difference equations. In this model, the decision rule maps the current state, denoted as $\psi_t = \{b^d_t, A_t, g_t\}$, into the value of government debt, $b_t$. The mapping is denoted as $b_t = f^b(\psi_t)$.

The complete model consists of a system of nonlinear equations including the first-order conditions from the household’s maximization problem, Equations (26) and (27); the government budget constraint, Equation (21); the specifications of the policy process and default scheme, Equations (22) and (3); the aggregate resource constraint, Equation (18); the specifications of shock processes, Equation (19) and (20); and the transversality condition, Equation (29). Appendix D shows the complete list of equations.

After substituting into the conjectured rule, the complete model can be reduced to a single equation:

$$
\frac{b^d_t + g_t + S(\psi_t) - \tau(\psi_t)A_t(1 - L(\psi_t))}{f^b(\psi_t)} = \beta E_t \left\{ \left( 1 - \Delta(f^b(\psi_t), A_{t+1}, g_{t+1}, b^*_t) \right) \frac{u_c(f^b(\psi_t), A_{t+1}, g_{t+1}, b^*_t)}{u_c(\psi_t)} \right\}
$$

(31)

The expectation in the right-hand side is evaluated using numerical quadrature. Equation (31) is solved for each set of state variables ($\psi$) defined over a discrete partition of the state space. The decision rule, $f^b(\psi)$, is updated at every node in the state space. The procedure is repeated until iterations update the current decision rule by less than some $\epsilon > 0$ (set to $1e-8$).

After finding the decision rule for government bonds, $f^b(\psi)$, I can solve the pricing rule, $q = f^q(\psi)$ using the government budget constraint, Equation (21). The interest rate on government bonds can also be solved using $R_t = \frac{1}{q_t}$, denoted as $f^R(\psi)$.

Calibration: The benchmark model is calibrated to the economy of Ireland. Table (2) concludes all the parameter specifications.

I take the model to be at an annual frequency. The household discount rate is set to be 0.95 and the net interest rate is 5.26%. The utility function is assumed to be $u(c, L) = \log c + \phi \log L$. The leisure preference parameter, $\phi$, is calibrated in such a way that the household spends 25% of time working. The total amount of time and the productivity at steady state are normalized to 1.

Note the state variable $b^d_t = (1 - \Delta_t)b_{t-1}$ incorporates 2-dimension information: the default threshold at time $t$, $b^*_t$, and the pre-default government liability, $b_{t-1}$.
The tax rate in steady state is 33%, lump-sum transfers are 13% of GDP, and government purchases are 17% of GDP. Table 1 lists the average of the average tax rates, lump-sum transfer over GDP ratios, and government purchases over GDP ratios across OECD countries from 1970 to 2008. The degree of countercyclical transfers ($\zeta$) and the tax adjustment parameter ($\gamma$) are estimated following the method in van den Noord (2000) and Girouard and Andre (2005). The estimated $\zeta$ is 1.86, using the data of detrended real net transfers and detrended real output per worker. The estimated $\gamma$ is 0.42, using the data of average tax rates and the debt over GDP ratios.

Parameters of shock processes are estimated for Ireland from 1970 to 2008. The productivity shock is estimated using detrended data of real GDP per worker and the government spending shock is estimated using detrended real government spending. Using HP filter, the productivity shock has persistence of 0.676 and standard deviation of 0.0223; and the government purchase shock has 0.69 and 0.0233 respectively.

The distribution of fiscal limit is constructed using MCMC simulation, explained in Section 3.1. I run 2000 simulations. Under the benchmark calibration, the mean of the distribution is 0.2471, around 99% of GDP; the standard deviation, $\sigma^b$, is 0.015. The default rate ($\delta$) is a policy choice variable. It can not be observed unless the sovereign actually defaults. In the recent debt crises, emerging market economies usually defaulted only part of their debt and, after debt renegotiations, agreed to honor a positive portion of the defaulted debt. Yue (2008) lists that the ratios of the defaulted debt over output vary from 10% in Pakistan to 32% in Argentina and the debt recovery rates vary from 100% in Pakistan and Ukraine to 28% in Argentina. This paper focuses on OECD countries where sovereign defaults are rare. I choose $\delta = 0.1$ so that the size of risk premium in my model can match the data. Nevertheless, the key result that sovereign risk premia arises nonlinearly with respect to the level of government debt is robust to different assumptions. An extension in Section 5 allows $\delta$ to depend on the state of the economy.

---

14 All data is from Economic Outlook No.84 from OECD source unless otherwise specified. The average tax rate is defined as total tax revenue (including social security tax, indirect tax and the direct tax) over GDP. Lump-sum transfers are defined as the sum of social security payment, net capital transfer and subsidy. Government purchases are defined as the sum of government final consumption and consumption of fixed capital.

15 van den Noord (2000) and Girouard and Andre (2005) estimated the size of automatic stabilizers by finding the elasticity of unemployment benefits with respect to the output gap.

16 Both real transfers and real output per worker are detrended using HP filter. The data of real GDP per worker is from Penn World Trade Table (2006).

17 The estimated persistences and standard deviations depend on the detrending method. Under linear detrending, both shocks have higher persistence and larger standard deviations: the productivity shock has persistence of 0.928 and standard deviation of 0.0320; and the government purchase shock has 0.9542 and 0.0323 respectively. Since the linear detrended data leads to a more dispersed distribution of fiscal limit and work in favor of our results, we bind our hands by calibrating to HP filtered data.
3.3 Result

This section discusses the decision rules and nonlinear simulations of the benchmark model.

**Decision rules:** The pricing rule of the interest rate of the government bond maps a 3-dimension state space to a 1-dimension state space, \( R_t = f_R(b^t_d, A_t, g_t) \). The rule is a 4-dimension surface in a single graph. For simplicity, we have plotted slices of the surface by fixing two variables at a time.

By keeping the government purchases \((g_t)\) at the steady state, the top panel of Figure (7) compares the response of net interest rate to the current government liability \((b^t_d)\) under different values of productivity \((A_t)\). The two green vertical lines, labeled \(b^*\) and \(b^{2std}\), are the mean of the fiscal limit and the threshold that is lower than the mean by two standard deviations \((b^{2std} = b^* - 2\sigma_b)\), respectively. The dashed red, solid blue, and dashed-dotted black line represent the responses of the interest rate when the level of productivity is low \((A_{min})\), at steady state, and high \((A_{max})\), respectively. First, the sovereign default risk premium rises with government liability in a nonlinear way. In all three cases the interest rate rises sharply as government liability enters the default regime, i.e. \(b^t_d\) is larger than \(b^{*2std}\). The financial market starts to demand a premium over the government bond as the probability of sovereign default begins to emerge from nil. Second, the level of productivity has a substantial impact on default risk premia. For a given level of current government liability, the lower the level of productivity, the higher the interest rate, i.e. the dashed red line lies above the dashed-dotted black line. In a recession, tax revenue is slashed and the government has to issue more debt in order to finance its expenditures. A higher debt level can substantially raise the sovereign default probability and risk premium of the government bond.

The bottom panel of Figure (7) compares the responses of the interest rate to the current government liability under different levels of government purchases \((g_t)\) while keeping the level of productivity \((A_t)\) at the steady state. Again, the interest rate rises sharply as the government liability enters the default regime. However, the interest rates are close as the government purchases vary, i.e. the dashed, solid and dashed-dotted lines are close. Government purchases affect its budget in two opposite directions. Higher expenditures drive up government liability. On the other hand, they crowd out private consumption and lead households to work more due to an income effect. The higher output raises tax revenues and helps to finance government purchases. Overall, different government purchases do not have a significant impact on the risk premium in this model.

**Simulation under benchmark calibration:** Figure (8) illustrates the simulation in which we draw a sequence of government spending and productivity shocks. Setting \(T = 540\), we
discard the first 500 periods in order to ensure that the model has settled into its steady state. Starting at the period $t = 510$, the economy receives shocks on government purchases and the level of productivity. The paths of output and government spending are simulated in such a way that they are close to the projection data from the Irish government and OECD Source, shown in Table (3). It illustrates the countercyclical fiscal policies in Ireland in 2009 economic crisis as government budgets deteriorate from strong countercyclical fiscal policies in a severe recession.

In Figure (8), lower wages from lower productivity discourage people from working and consuming. Lower tax revenue, due to lower labor supply and productivity, and higher government spending force the government to borrow more. Interest rates rise partly because households substitute away from future consumption and partly because government liability approaches the fiscal limit, i.e. the green line $b^*_t$, which is randomly drawn from the distribution $\mathcal{N}(b^*, \sigma^2_b)$. Rising interest payments further deteriorate the government budget.

In order to disentangle the effect of the intertemporal substitution from the risk premium in the rising interest rate, Figure (8) compares the benchmark model with a default-free economy in which the sovereign bond is not constrained by the fiscal limit and $\delta$ is zero. The dashed blue lines represent the benchmark model, while the solid black lines represent the default-free economy under the same sequences of shocks. The risk premium of one-period bond, defined as the interest rate difference between the two economies, is around 80 basis points for a prolonged period.

**Long-term bond:** In practice, long-term bond spreads rather than short-term bond spreads are used to measure sovereign default risk premia. An $n$-period bond can be priced as,

$$ Q^n_t = \beta^n E_t \left( (1 - \Delta_{t+n}) \frac{u_c(t+n)}{u_c(t)} \right). $$

The multiple integration is calculated using MCMC simulation.\(^{18}\)

Figure (9) compares the default risk premia of 1-year and 10-year bonds under different states of the economy. The top panel shows that the premium of the long-term bond jumps ahead of the short-term bond and provides early warnings of sovereign defaults in a recession. In booms, however, the long-term bond provides insurance and has much lower default premium than the short-term bond for the same level of government debt, shown in the middle panel. In steady state, the default risk premium of the long-term bond emerges at a lower level of government debt but rises at a slower pace, comparing to the short-term bond.

Conditional on the same shocks as in Figure (8), Figure (10) illustrates that the long-term bonds give advance warnings of sovereign defaults in a severe recession. Expected high

\(^{18}\)It is challenging to handle off-grid points in a highly nonlinear model as the one we have here. We use finite-element method to approximate the decision rules of one-period bond.
government indebtedness in the future results in a rise in current risk premia of long-term bonds, even the risk premia of short-term bonds are flat. The longer the bond maturity, the earlier the default risk premia start to emerge.

**Simulations under Alternative Calibrations:** Section 3.1 explains that the distribution of fiscal limit depends on parameters like political uncertainty, degree of the countercyclical transfer policy and the size of the government. Figure (11) illustrates how those parameters affect the dynamics of the economy. The left panels compare the responses of government bond and net interest rate under different political uncertainties. Dashed blue lines represent the benchmark case where the political factor is small ($\theta = 0.937$) and the political uncertainty is high, while solid black lines represent an economy where the political factor is large ($\theta = 1$) and the political uncertainty is low. Since it leads to a lower mean of the distribution of stochastic fiscal limit, high political uncertainty generates substantial risk premia over extended periods.

In the middle panels, dashed blue lines show the responses of government bond, net interest rate and transfers under strong countercyclical transfer policy ($\zeta = 1.86$), while solid lines represent an economy with fixed transfers ($\zeta = 0$). Strong countercyclical fiscal policy deteriorates the government budget and raises interest rate over a prolonged period. The right panels compare two economies with different government size: the ratio of transfers over GDP is 5.5% in solid blue lines and 13.5% in dashed blue lines. The size of the government affects the mean of the distribution of stochastic fiscal limit and, therefore, the risk premia of government bond.

4 Discussion of Fiscal Policy

Section 3 develops an infinite-horizon model with endogenous fiscal limit. The framework is flexible enough to be adopted in a broader analysis. This section will use the infinite-horizon model to discuss the impact of different fiscal policies on the economy.

**Different tax adjustment parameter:** The willingness of the government to retire sovereign debt by raising tax rates changes over time and varies across countries. Figure (12) compares two economies with different fiscal adjustment parameters: $\gamma$ is 0.42 or 0.7. As the level of the debt rises by 10%, the tax rate rises by 1% if $\gamma$ is 0.42 and by 1.7% if $\gamma$ is 0.7. Given the same shocks, faster fiscal adjustments prevent the economy from slipping into the default regime and from paying the risk premium. In the economy with slow fiscal adjustments, relatively low tax rates allow households to work and consume more in the early periods. However, the sovereign debt builds up over time and eventually approaches the default regime. Private consumption and labor supply will, thus, be lower over a prolonged period.
Regime-switching Government Purchases: Regime-switching government purchases are frequently observed in OECD countries. The comparison of Italy and Belgium well illustrates this point. As sovereign debt was soaring in both countries in 1980s, people expected both governments to cut spendings with some probability. Only Belgium lived up to that expectation, shown in Figure (4).

In this experiment, the government purchases follow a regime-switching rule, instead of being an exogenous AR(1) process. It is constant and stays at its steady state level in regime 1. In regime 2, it adjusts according to the level of government liability with a fiscal adjustment parameter of $\alpha$. A larger $\alpha$ implies that the government is more willing to retire debt by cutting government purchases. The switch between the two regimes follows a Markov process:

$$g_t - g = \begin{cases} 
0 & \text{in Regime 1} \\
-\alpha(b^d_t - b) & (\alpha > 0) \text{ in Regime 2}
\end{cases}$$

with transition probability $P$:

$$\begin{Bmatrix}
p_1 & 1 - p_1 \\
1 - p_2 & p_2
\end{Bmatrix}$$

$\alpha$ is calibrated to 0.2 so that the government cuts its purchases by 2% of the GDP if its debt rises by 10% of the GDP. The probability of staying in the same regime is set to 0.9 ($p_1 = p_2 = 0.9$), reflecting the enormous political difficulty to change the government spending in a systematical manner in general.

We simulate two dynamic paths under two regimes conditional on the transition probability $P$. In Figure (13), the solid black lines represent regime 1 where government purchases are always at steady state level, and the dashed blue lines represent regime 2 where government purchases are cut as the sovereign debt soars. Two observations emerge from the comparison. First, the interest rate is higher in regime 2 than regime 1 in the early periods, as household expects lower tax rate in the future. Second, the economy returns to its steady state much faster under regime 2 than under regime 1. If the government purchases do not respond to the sovereign debt, rising debt has to be financed by higher tax rate and higher borrowing. It eventually pushes the level of the debt to the default regime.

5 Robustness: State-dependent Default

This section extends the basic model specified in Section 3 by allowing the default rate, $\delta$, to be proportional to the level of the debt, instead of being fixed. At default, the government reneges whatever amount it needs to bring the level of sovereign debt to $b^*_{2\text{ std}}$. Since sequential defaults are rare in the data, this experiment is designed in such a way that the government doesn’t default in sequential multiple periods in general.

$$(1 - \Delta_t)b_{t-1} = \begin{cases} 
b_{t-1} & \text{if } b_{t-1} < b^*_t \\
b^*_{2\text{ std}} & \text{if } b_{t-1} \geq b^*_t
\end{cases}$$
Equivalently,
\[
\Delta_t = \begin{cases} 
0 & \text{if } b_{t-1} < b_t^* \\
\delta_t = 1 - \frac{b_{2\text{std}}}{b_{t-1}} & \text{if } b_{t-1} \geq b_t^*
\end{cases}
\]

Since \(\delta_t\) is positively correlated with \(b_{t-1}\), the higher the level of government debt, the higher the time-varying default rate.

Figure (14) compares the pricing rules under endogenous default rate (\(\delta_t\)) and fixed default rate (\(\delta\)). The rules are virtually identical when the government liability (\(b^d\)) is low and the default probability is close to zero. If the liability is at the low end of default regime (\(b^* > b^d > b^*_2\text{std}\)), the interest rate is higher in the economy with fixed default rate as the endogenous default rate (\(\delta_t\)) depends on the level of government liability. As the level of government liability surges to the high end of default regime (\(b^d > b^*\)), the endogenous default rate can be higher than the fixed rate (\(\delta\)) by a substantial amount and the interest rate, therefore, is higher in the economy with endogenous default rate.

6 Conclusion

This paper presents a general equilibrium framework in which the Laffer Curve, implied by distortionary taxation, limits the government’s ability to service its debt. The two-period model highlights the channel through which the underlying macroeconomic factors affect the government’s ability to pay and, therefore, the sovereign default risk premia. This channel remains at work in the infinite-horizon model, in which a stochastic fiscal limit arises endogenously from the dynamic Laffer Curves. We solved the infinite-horizon model numerically using monotone mapping method.

The main messages can be summarized as following. First, sovereign default risk premia hinge on the size of the government, the degree of the countercyclical fiscal policy, economic diversity and political uncertainty. Second, the infinite-horizon model produces a nonlinear relationship between the default risk premia and the level of government debt. The risk premia start to emerge as the level of debt approaches the fiscal limit. In addition, long-term bonds give early warnings of sovereign defaults in recessions. The longer the bond maturity, the earlier the default risk premium emerges. Finally, if the government is flexible in that it can raise the tax rate or cut government expenditures (or both), then it can steer through hard times without jeopardizing its sovereign rating.
References


Appendix A: Solving the Two-Period Model

**Laffer Curve:** Given the quasi-linear preference, the labor supply is,

\[ L_t = \frac{\sigma}{A_t(1 - \tau_t)}. \]  

(33)

The tax revenue is,

\[ T_t = A_t\tau_t(1 - L_t) = \tau_t(A_t(1 - \tau_t) - \sigma), \]  

(34)

Take the derivative of \( T_t \) with respect to \( \tau_t \),

\[ \frac{\partial T_t}{\partial \tau_t} = A_t - \frac{\sigma}{(1 - \tau_t)^2}, \]  

(35)

Therefore, the total tax revenue peaks at \( T_{t}^{\text{max}} \) as the tax rate is \( \tau_{t}^{\text{max}} \),

\[ T_{t}^{\text{max}} = \sqrt{A_t} \left( \sqrt{A_t} - \sqrt{\sigma} \right)^2 \]  

(36)

\[ \tau_{t}^{\text{max}} = 1 - \sqrt{\frac{\sigma}{A_t}} \]  

(37)

**The solution:** Due to the quasi-linear preference, the bond price is,

\[ q_1 = \beta E_1 (1 - \Delta_2) \]  

(38)

Given \( A_1 = A_L \), the level of productivity in period 2 is,

\[ A_2 = \begin{cases} A_H & \text{with probability of } 1 - p \\ A_L & \text{with probability of } p \end{cases} \]

The government will pay back its debt unless \( A_2 \) is low and it hits the economic fiscal limit. The budget constraint in period 2 will have the following possibilities,

\[ \begin{cases} \bar{S} = A_H\tau_2(1 - L_2) - b_1 & \text{with probability of } 1 - p \\ \tilde{S} = T_{L_t}^{\text{max}} - (1 - \Delta_2)b_1 & \text{with probability of } p \end{cases} \]

Therefore, the bond price is,

\[ q_1 = \beta(1 - p) + \beta p \left( q_1 \frac{T_{L_t}^{\text{max}} - \tilde{S}}{\bar{S} - T_1} \right) \]  

or \[ q_1 = \frac{\beta(1 - p)}{1 - \beta p \frac{T_{L_t}^{\text{max}} - \tilde{S}}{\bar{S} - T_1}} \]  

(39)
Extension with political uncertainty: We assume that even if \( A_2 = A_H \), the government might choose not to pay back its debt with the probability of \( 1 - \theta \).

\[
q_1 = \beta (1 - p) + \beta (1 - \theta)(1 - p) \left( q_1 \frac{T_H^2 - \bar{S}}{S - T_1} \right) + \beta p \left( q_1 \frac{T_{L_{\text{max}}}^H - \bar{S}}{S - T_1} \right)
\]

or

\[
q_1 = \frac{\beta (1 - p) \left( 1 - \beta p \frac{T_{L_{\text{max}}}^H - \bar{S}}{S - T_1} - \beta (1 - \theta)(1 - p) \frac{T_H^2 - \bar{S}}{S - T_1} \right)}{1 - \beta p \frac{T_{L_{\text{max}}}^H - \bar{S}}{S - T_1} - \beta (1 - \theta)(1 - p) \frac{T_H^2 - \bar{S}}{S - T_1}}
\]  

(40)

where \( T_H^2 \) is the tax revenue in period 2 given \( A_2 = A_H \) and \( \tau_2 = \bar{\tau} \).

**Appendix B: Derivation of Equation (29)**

From the household’s maximization problem, the first-order condition is:

\[
q_t = \beta E_t (1 - \Delta_{t+1}) \frac{u_c(t+1)}{u_c(t)}
\]  

(41)

The transversality condition is:

\[
\lim_{j \to \infty} E_t \beta^{j+1} q_t q_{t+1} \cdots q_{t+j} b_{t+j} = 0
\]  

(42)

Substitute the first-order condition, Equation (41), into the transversality condition, Equation (42):

\[
\lim_{j \to \infty} E_t \beta^{j+1} q_t q_{t+1} \cdots q_{t+j} q_{t+j} b_{t+j} = 0
\]  

(43)

It is equivalent to:

\[
\lim_{j \to \infty} \beta^{j+1} E_t \frac{u_c(t+j+1)}{u_c(t)} \left( 1 - \Delta_{t+j+1} \right) b_{t+j} = 0
\]  

(44)

**Appendix C: MCMC Simulation of Fiscal Limit**

In the infinite-horizon model, household consumption and labor supply only depend on the income tax rate, \( \tau_t \), and the exogenous state variables, \( \{A_t, g_t\} \). Assume the utility function is \( u(c, L) = \log c + \phi \log L \). Household first-order conditions can be written as,

\[
1 - L_t = \frac{A_t (1 - \tau_t) + \phi g_t}{A_t (1 + \phi - \tau_t)}
\]

\[
c_t = \frac{(A_t - g_t)(1 - \tau_t)}{1 + \phi - \tau_t}
\]  

(45)
Therefore, the tax revenue is,

\[ T_t = \tau_t A_t (1 - \tau_t) + \phi g_t \frac{1}{1 + \phi - \tau_t} \]

\[ = \frac{(1 + 2\phi) A_t - \phi g_t}{A} - \left( \frac{A_t (1 + \phi - \tau_t) + \left( \frac{1 + \phi}{1 + \phi - \tau_t} \right) C}{B x_t} \right) \]

\[ \equiv A - \left( B x_t + \frac{C}{x_t} \right) \]

(46)

The tax revenue reaches to \( T_{t\text{max}} \) if \( \tau_t = \tau_{t\text{max}} \). It can be shown that:

\[ \tau_{t\text{max}} = 1 + \phi - \sqrt{\frac{C}{B}} \]

\[ = 1 + \phi - \sqrt{(1 + \phi)\phi(A_t - g_t)} / A_t \]

(47)

\[ T_{t\text{max}} = A - 2\sqrt{BC} \]

\[ = (1 + 2\phi) A_t - \phi g_t - 2(1 + \phi)\phi A_t (A_t - g_t) \]

(48)

The peak point of Laffer Curve only depends on exogenous shock \( A_t \) and \( g_t \). It implies that there exists a mapping between the state space \((A, g)\) to \( \tau_{t\text{max}} \), denoted as \( \tau_{t\text{max}}(A, g) \). Using the first-order conditions of household and the aggregate resource constraint, I can also find the mappings of consumption and leisure, \( c_{t\text{max}}(A, g) \) and \( L_{t\text{max}}(A, g) \).

**MCMC procedure:** Numerically, I find the distribution of \( b^*_t \) using MCMC simulation, and approximate it as a normal distribution \( N(b^*, \sigma_b^2) \).

- Given the state at the starting point \((A, g)\), I simulate a path \((A^{(1)}_h, g^{(1)}_h)\) with random draws from the shock processes with \( h = 1, ..., H \).

- Given the mappings of \( \tau_{t\text{max}}(A, g) \), \( c_{t\text{max}}(A, g) \) and \( L_{t\text{max}}(A, g) \), I can find \( \tau_{t\text{max}}^h \), \( c_{t\text{max}}^h \) and \( L_{t\text{max}}^h \) at each point of the path, and therefore find \( h^{(1)} \) conditional on the specific path of \((A^{(1)}_h, g^{(1)}_h)\) with \( h = 1, ..., H \).

- Again, simulate another path \((A^{(2)}_h, g^{(2)}_h)\) with random draws of shocks at each point \( h \). Repeat the above two steps and find \( b^{(2)} \) conditional on the specific path of \((A^{(2)}_h, g^{(2)}_h)\) with \( h = 1, ..., H \).

- I can simulate \( N \) paths and approximate the distribution as \( N(b^*, \sigma_b^2) \).
Appendix D: Equations of the Benchmark Model

\[
\frac{u_L(t)}{u_c(t)} = 1 - \tau_t
\]

\[
q_t = \beta E_t(1 - \Delta_{t+1}) \frac{u_c(t+1)}{u_c(t)}
\]

\[
\tau_t - \tau = \gamma \left( b_t^d - \bar{b} \right)
\]

\[
\ln \frac{S_t}{S} = -\zeta \ln \frac{A_t}{A}
\]

\(\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b_t^* \\ \delta & \text{if } b_{t-1} \geq b_t^* \end{cases}\)

\[
b_t^* = E_t \sum_{h=0}^{H} \beta^h \frac{u_c^{\text{max}}(t+h)}{u_c^{\text{max}}(t)} (T_{t+h}^{\text{max}} - g_{t+h} - S_t)
\]

\[
\tau_t A_t (1 - L_t) + b_t q_t = (1 - \Delta_t)b_{t-1} + g_t + S_t
\]

\[
c_t + g_t = A_t (1 - L_t)
\]

\[
\ln \frac{A_t}{A} = \rho_u \ln \frac{A_{t-1}}{A} + u_t \quad u_t \sim \mathcal{N}(0, \sigma_u^2)
\]

\[
\ln \frac{g_t}{g} = \rho_e \ln \frac{g_{t-1}}{g} + e_t \quad e_t \sim \mathcal{N}(0, \sigma_e^2)
\]

\[
\lim_{j \to \infty} \beta^{j+1} E_t \frac{u_c(t+j+1)}{u_c(t)} \left( (1 - \Delta_{t+j+1})b_{t+j} \right) = 0
\]
Table 1: Average Fiscal Ratios across OECD Countries (1970 - 2008)

<table>
<thead>
<tr>
<th></th>
<th>τ</th>
<th>S/Y</th>
<th>g/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td>26.39</td>
<td>9.67</td>
<td>16.22</td>
</tr>
<tr>
<td>US</td>
<td>27.16</td>
<td>10.5</td>
<td>18.61</td>
</tr>
<tr>
<td>Switzerland</td>
<td>27.97</td>
<td>15.65</td>
<td>13.63</td>
</tr>
<tr>
<td>Greece</td>
<td>28.09</td>
<td>13.25</td>
<td>19.8</td>
</tr>
<tr>
<td>Australia</td>
<td>28.15</td>
<td>8.47</td>
<td>19.8</td>
</tr>
<tr>
<td>Spain</td>
<td>29.95</td>
<td>13.39</td>
<td>17.01</td>
</tr>
<tr>
<td>NZ</td>
<td>30.90</td>
<td>13.36</td>
<td>20.1</td>
</tr>
<tr>
<td>Canada</td>
<td>32.38</td>
<td>11.78</td>
<td>22.95</td>
</tr>
<tr>
<td>Ireland</td>
<td>32.73</td>
<td>12.94</td>
<td>17.52</td>
</tr>
<tr>
<td>UK</td>
<td>36.28</td>
<td>14.83</td>
<td>21.45</td>
</tr>
<tr>
<td>Italy</td>
<td>36.49</td>
<td>17.68</td>
<td>19.94</td>
</tr>
<tr>
<td>Germany</td>
<td>40.46</td>
<td>20.9</td>
<td>26.02</td>
</tr>
<tr>
<td>Netherlands</td>
<td>41.03</td>
<td>17.74</td>
<td>22.33</td>
</tr>
<tr>
<td>Finland</td>
<td>41.86</td>
<td>17.41</td>
<td>24.19</td>
</tr>
<tr>
<td>France</td>
<td>42.20</td>
<td>18.84</td>
<td></td>
</tr>
<tr>
<td>Austria</td>
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<td>22.37</td>
<td>20.37</td>
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<td>Norway</td>
<td>42.56</td>
<td>17.11</td>
<td>21.99</td>
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<tr>
<td>Belgium</td>
<td>43.34</td>
<td>18.71</td>
<td>23.76</td>
</tr>
<tr>
<td>Denmark</td>
<td>46.72</td>
<td>20.95</td>
<td>27.48</td>
</tr>
<tr>
<td>Sweden</td>
<td>48.35</td>
<td>19.1</td>
<td>28.85</td>
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Data source: OECD source (2008)
Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Benchmark</th>
<th>Ireland (1970-2008)</th>
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<tbody>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.95$</td>
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<tr>
<td>Political Discount factor</td>
<td>$\theta = 0.9368$</td>
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<td>Leisure</td>
<td>$L = 0.25$</td>
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<tr>
<td>Government purchase-GDP</td>
<td>$g/y = 0.17$</td>
<td>17.52%</td>
</tr>
<tr>
<td>Transfer-GDP</td>
<td>$S/y = 0.13$</td>
<td>12.94%</td>
</tr>
<tr>
<td>Automatic stabilizer</td>
<td>$\zeta = 1.86$</td>
<td>1.86</td>
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<tr>
<td>Tax rate</td>
<td>$\tau = 0.33$</td>
<td>32.73%</td>
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<tr>
<td>Tax adjustment parameter</td>
<td>$\gamma = 0.42$</td>
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<tr>
<td>Persistence of $A$ shock</td>
<td>$\rho^u = 0.676$</td>
<td>0.676 (HP Filter)</td>
</tr>
<tr>
<td>Standard deviation of $A$ shock</td>
<td>$\sigma^u = 0.0223$</td>
<td>0.0223</td>
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<tr>
<td>Persistence of $g$ shock</td>
<td>$\rho^e = 0.69$</td>
<td>0.69 (HP Filter)</td>
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<tr>
<td>Standard deviation of $g$ shock</td>
<td>$\sigma^e = 0.0233$</td>
<td>0.0233</td>
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<td>Default rate</td>
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Table 3: Shock Parameters

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<tr>
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<th>2010</th>
<th>2011</th>
<th>2012</th>
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<td>-8.7</td>
<td>-6.86</td>
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<tr>
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<td>-6.3</td>
<td>-8.2</td>
<td>-6.8</td>
<td>-4.3</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>18.41</td>
<td>20.41</td>
<td>21.28</td>
<td>20.92</td>
<td>20.22</td>
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<tr>
<td>Data</td>
<td>18.41</td>
<td>20.57</td>
<td>21.36</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Note: Output gap data is from Ireland Department of Finance (2009); data of government spending-GDP ratios are from OECD Source (2008).
Figure 1: Sovereign Rating across OECD Countries (Standard & Poor’s)
Figure 2: Net Debt-GDP Ratio and Sovereign Rating: U.K. vs. U.S.
Figure 3: Net Debt-GDP Ratio and Sovereign Rating: New Zealand vs. Canada
Figure 4: Net Debt-GDP Ratio and Sovereign Rating: Italy vs. Belgium
Figure 5: Italy vs. Belgium (top panels: net debt-GDP ratios (dashed blue lines) vs. government spending and net transfer over GDP ratios (solid green lines); bottom panels: net debt over GDP ratios (dashed blue lines) vs. tax rate (solid green lines).)
Figure 6: Determinants of Distribution of $\frac{v'}{v}$: top left panel: distribution in the benchmark calibration to Ireland; middle left panel: political discount factors ($\theta$); bottom left panel: countercyclical transfer policies ($\zeta$); top right panel: government size; middle right panel: shock persistence ($\rho$); bottom right panel: shock variance ($\sigma^2$).
Figure 7: Pricing rule of net interest rate (top panel: $r(b) = R(b) - 1$ at a high, steady-state, and low level of $A$ while $g$ is in steady state; bottom panel: $r(b)$ at a high, steady-state, and low level of $g$ while $A$ is in steady state.)
Figure 8: Responses to a severe recession (calibrated to projection data of Ireland from 2008 to 2012). Under the same shocks, dashed blue lines represent a stochastic default scheme ($\delta = 0.1$) and solid black lines represent a default-free scheme ($\delta = 0$). Time units in years.
Figure 9: Pricing rules of default risk premia with different maturities: solid blue lines represent 1-year bond, while dashed red lines show 10-year bond. The relative risks depend on the state of the economy.
Figure 10: Risk premia of long-term bonds with different maturities using MCMC simulation, conditioning on the shocks in Figure (8).
Figure 11: Responses to the same shocks under different calibrations: left panels compare different political discount factors; middle panels compare different countercyclical transfer policies; right panels compare different government sizes. Time units in years.
Figure 12: Responses to a severe recession (calibrated to projection data of Ireland from 2008 to 2012). Under the same shocks, solid black lines represent the benchmark case ($\gamma = 0.42$); dashed blue lines represent a larger tax adjustment parameter ($\gamma = 0.7$). Time units in years.
Figure 13: Regime-Switching Government Purchase Model: under the same shocks, the blue dashed lines represent the economy in which the government reduce government purchases, while the solid black lines represent the economy in which the government does not change its purchase.
Figure 14: Decision rule of net interest rate ($r(b) = R(b) - 1$) as both states ($g$ and $A$) are at steady state: dashed blue lines represent endogenous default scheme; solid red lines represent exogenous default scheme.