

Transmission Mechanisms of the Public Debt

Tack Yun

Department of Economics

Seoul National University

This Draft: October, 2011

Abstract

The first part of this paper is devoted to present some empirical evidence on the empirical IS curve that relates the aggregate demand to the real anticipated interest rate and the ratio of the public debt to GDP. The result of the empirical analysis is in favor of the presence of an empirical IS curve in which the aggregate demand is significantly affected by both the real anticipated interest rate and the public debt-to-GDP. In particular, the empirical finding on negative debt-elasticities of the aggregate demand in the empirical IS curve suggests the modification of the criteria used for the conventional distinction between non-Ricardian and Ricardian regimes. While the conventional distinction tells that the Taylor principle delivers an explosive dynamics of equilibrium path in a non-Ricardian regime with an exogenous primary surplus process, the estimated empirical IS curve indicates that a unique local equilibrium path requires the Taylor principle even with non-Ricardian regime in the presence of negative debt-elasticities of the aggregate demand. Furthermore, since the main modelling point of this paper is that the IS curve is the key equation for the transmission of the public debt, a goal of this paper is to see a variety of alternative theoretic mechanisms that might help include the public debt into the IS curve. As a result, a set of different models are discussed in this paper: Sovereign-risk based channel or Risk-premium (financial friction) based channel, Modified liquidity-effect channel, and Market-segmentation (in market for government securities) channel.

JEL classification: E20; E31; E32; H30; H60

Keywords: Empirical IS Curve; Fiscal Transmission Mechanism; Fiscal-Monetary Interaction; Public Debt

E-mail address: tackyun@snu.ac.kr.

A part of this paper has been spawned by my discussion of “Fiscal Policy and Financial Stability” by Philip Lane at the International Conference of the Bank of Korea. I appreciate kind encouragement and helpful discussion of Thomas Sargent, Eric Leeper, and Todd Walker for making this paper. All errors are my own.

1 Introduction

The recent research on the fiscal theory of the price level determination has presented multiple channels through which changes in the public debt can affect the aggregate inflation. Changes in the level of the public debt might also affect the aggregate demand through various channels: Learning based channel, Sovereign-risk based channel, Risk-premium (financial friction) based channel, Liquidity-effect channel, Wealth-effect channel, Portfolio-balance channel, and Market-segmentation (in market for government securities) channel.

The first part of this paper is devoted to present some empirical evidence on an empirical IS relation that connects the aggregate demand to the real anticipated interest rate and the ratio of the public debt to GDP. The result of the empirical analysis is significantly in favor of the presence of an empirical IS curve in which the aggregate demand is affected by both the real anticipated interest rate and the public debt-to-GDP.

In particular, the empirical finding on negative debt-elasticities of the aggregate demand in the empirical IS curve suggests modification of the criteria used for the conventional distinction between non-Ricardian and Ricardian regimes. While the conventional distinction tells that the Taylor principle delivers an explosive dynamics of equilibrium path in a non-Ricardian regime with an exogenous primary surplus process, the estimated empirical IS curve indicates that a unique local equilibrium path requires the Taylor principle even with non-Ricardian regime in the presence of negative debt-elasticities of the aggregate demand.

The theoretic motivation behind the empirical analysis of this paper can be explained as follows. Although many different transmission channels potentially exist, it is possible to partition them into distinct equivalence classes (up to the first-order approximation) within a small structural-model framework, especially by using the IS curve. In particular, if the public debt plays a role in the inter-temporal optimization problems of households, a canonical representation of the resulting IS curve can be written as

$$x_t = \sigma_x E_t[x_{t+1}] - \sigma_r (r_t - E_t[\pi_{t+1}]) + \sigma_b \hat{b}_t$$

where x_t denotes the output gap, r_t is the nominal interest rate, π_t is the inflation rate, \hat{b}_t is the real public debt, and coefficients σ_x and σ_r are positive. But the sign of σ_b depends how the role of the public debt is incorporated into models. Specifically, the convenience yield of government securities and the incorporation of their wealth effect lead to a positive value for σ_b . The sovereign-risk or financial-friction based channels of the public debt can generate a negative value of σ_b . In addition, a learning based channel might take place when the presence of fiscal uncertainty and resulting debates on its consequence on inflation may lead agents not to be confident about the true determination of inflation expectations, while this possibility is already explored in Sims (2011)

and Leeper and Walker (2011).

A small-sized DSGE model is then used to demonstrate that the sign of the elasticity of the public debt in the IS equation plays an important role in determining whether or not the conventional wisdom for monetary-fiscal interactions breaks down. In the presence of risk-premium (financial friction) based channel, for example, it might be necessary to modify the criteria used for the conventional distinction between non-Ricardian and Ricardian regimes. Furthermore, since the IS curve is the key equation for the transmission of the public debt in this paper, a different transmission mechanism of the public debt corresponds to a different scenario for the inclusion of the public debt into the IS curve. Thus, a goal of this paper is to see a variety of alternative theoretic mechanisms that might help include the public debt into the IS curve. As a result, many different models are discussed in this paper. However, these models are not new in the literature.

The first one allows for the possibility of default in the government debt because expected future expenditures exceed expected future revenues. The default rate of government securities should be obtained by imposing the restriction that the expected present-value of the public debt at the indefinite future must be zero, following Uribe (2006) and Shabert (2010). The tentative result of this paper is that the introduction of sovereign default with having any financial friction might not create qualitatively substantial changes in the conventional dichotomy between non-Ricardian and Ricardian regimes.

But a slight modification of the market structure of government securities may help break down the conventional wisdom about the role of fiscal policy regimes in restoring the local uniqueness of equilibrium dynamics when monetary policy rules are chosen to move very mildly in response to a rise in the aggregate inflation. In order to show this result, an example model of this paper reflects the fact that primary and secondary markets exist for government securities, while participation restriction is imposed on the primary market. Given this type of market structure, participants of the primary market might earn positive profits by exploiting their monopoly status. For example, primary dealers can lend their funds to retail banks or sell government bonds in the secondary market at the interest rate committed by the government, while primary dealers trade within participants of the primary market at a discounted rate because of the future default risk. In this framework, the IS curve contains the public debt as one of its arguments when the future default rate is expected to be determined by the ratio of the public debt to GDP.

Alternatively, the negative debt-elasticities of the aggregate demand might reflect the possibility that an increase in the level of the public debt raises the spread of interest rates (including the one that is applied to households) from the policy rate of the central bank. In this vein, Garcia-Cicco, Pancrazi, and Uribe (2011) has discussed the role of financial frictions in the real business cycles

of emerging countries by incorporating a specification that enables econometrician to estimate the debt elasticity of a country's premium, rather than fixing it at a small number. In particular, their specification might be relevant when fiscal uncertainty affects financial friction (that can be measured in terms of interest-rate spreads) especially in the era of a persistently high level of the public debt.

The second channel arises in a modified version of the convenience yield of government securities, while the convenience yield of government securities might arise because of their liquidity and safety. The modification is made to allow for the possibility that investors might want to prefer a short maturity of government securities than a long maturity especially when they perceive a substantial amount of sovereign default risk. Specifically, marginal utility benefits of holding short-term government securities rise as the total size of the public debt increases. Given this modification, it is demonstrated that in the presence of the convenience yield of short-term government securities, the risk premium of long-term government securities rises as the ratio of the public debt to GDP rises even in the log-linear approximation of the model. In this model, keeping the Taylor principle helps guarantee the local uniqueness of equilibrium dynamics even when the real primary surplus is exogenously determined.

The third channel is closely associated with the framework of the market-segmentation hypothesis of the term structure of interest rates. For example, commercial banks take deposits from households to hold short-term government bonds and make loans to other financial institutions, while investment banks hold only long-term government securities in the asset side of their balance sheets. Because of this assumption for commercial and investment banks, investors are segmented between markets of short-term and long-term government securities.

The next section discusses an empirical relation between the aggregate demand and the ratio of the public debt to GDP. In section 3, a small DSGE model is analyzed to understand the implication of incorporating the public debt into the IS curve for the determinacy of the equilibrium dynamics. Sections 4 and 5 present business-cycle implications of the empirical IS curve with the public debt in normal times, along with its implications for the equilibrium dynamics under zero-bound on the nominal interest rate. The following several sections will give detailed discussions of several specific (small-scale) DSGE models in order to demonstrate that there are alternative ways to make the public debt included in the IS curve.

2 The Public Debt and the Empirical IS curve

In this section, the empirical relation between the aggregate demand and the ratio of the public debt to GDP is estimated within the framework of a structural relation between the aggregate

demand and the anticipated real interest. Specifically, an empirical IS curve is augmented with the ratio of the public debt to GDP, allowing for the impact of the public debt in the determination of the aggregate demand:

$$x_t = E_t[x_{t+1}] - \psi_r(r_t - E_t[\pi_{t+1}]) - \psi_b(\hat{b}_t - x_t) \quad (2.1)$$

where x_t denotes the output gap, r_t is the nominal interest rate, π_t is the inflation rate, \hat{b}_t is the real public debt, ψ_r measures the interest elasticity of the aggregate demand, and ψ_b is the debt elasticity of the aggregate demand. The estimation of this empirical relation would lead to a positive estimate of ψ_b when the aggregate demand is negatively associated with the real interest rate. In addition, a positive estimate of ψ_b would show up in the presence of the wealth effect or the convenience yield of the public debt, while a positive value of ψ_b implies that the aggregate demand might be affected by financial frictions or the possibility of the default risk of the public debt.

A notable feature of the aggregate demand curve specified above is that it permits both backward-looking and forward-looking specifications for the estimation of the empirical IS curve. For example, the empirical IS curve equation can be rewritten as

$$\Delta x_t = \psi_r(r_{t-1} - E_{t-1}[\pi_t]) + \psi_b(\hat{b}_{t-1} - x_{t-1}) + u_t \quad (2.2)$$

where Δx_t is the realized change of the aggregate demand between periods $t - 1$ and t , and $u_t (= x_t - E_{t-1}[x_t])$ denotes the expectation error for the current aggregate demand.

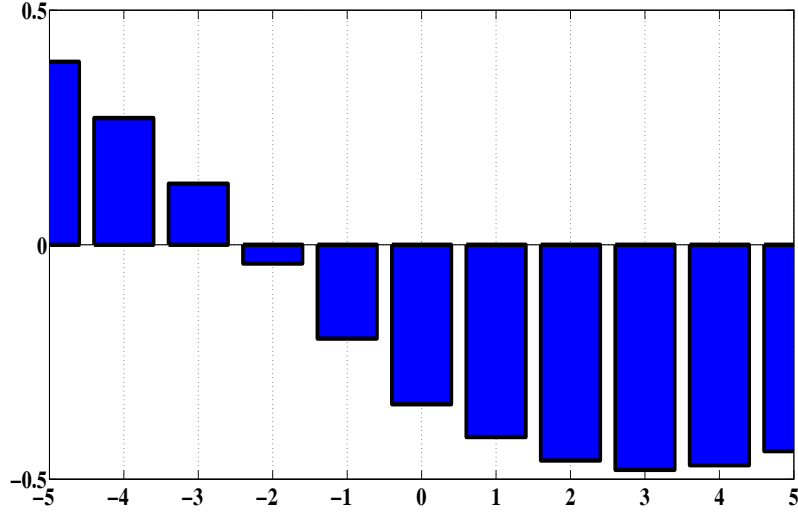
Table 1 reports the estimation results of the empirical IS curve described above. In this table, the first and second equations correspond to backward-looking specifications, while the other equations correspond to a forward-looking specification. The sample consists of the U.S. quarterly data on the output gap, anticipated real interest rate, and debt-to-GDP ratio that are observed during the period of 1981:Q3 - 2011:Q2. The output gap is defined as the deviation of the real GDP from its potential GDP (that is constructed by the Congressional Budget Office) and the real interest rate is the federal funds rate minus the expected inflation rate that is obtained from the Survey of Professional Forecasters (collected by the Federal Reserve Bank of Philadelphia). The debt to GDP is the ratio of the total (U.S.) government's debt held by the public to GDP and the debt to GDP ratio is detrended by using the Hodrick-Prescott filter. Moreover, coefficients of the first and second equations are estimated by using the method of ordinary least squares, while the generalized method of moments is used to estimate the forward-looking specification and the instrument variables include two-quarter and four-quarter lagged real interest rate, debt-to-GDP ratio, and output gap.

Table 1: Estimation Results of the Empirical IS Curve

Backward-looking specification									
Δx_t	=	0.15	$(r_{t-1} - E_{t-1}[\pi_t])$	+	0.20	$\hat{b}_{p,t-1}$			
		(0.10)			(0.03)				
x_t	=	0.12	$(r_{t-1} - E_{t-1}[\pi_t])$	+	0.21	$\hat{b}_{p,t-1}$	+	1.03	x_{t-1}
		(0.1)			(0.03)			(0.02)	
Forward-looking specification									
x_t	=	- 0.42	$(r_t - E_t[\pi_{t+1}])$	-	0.27	$\hat{b}_{p,t}$	+	1.01	$E_t[x_{t+1}]$
		(0.16)			(0.06)			(0.03)	
x_t	=	- 0.41	$(r_t - E_t[\pi_{t+1}])$	-	0.28	$\hat{b}_{p,t}$	+		$E_t[x_{t+1}]$
		(0.15)			(0.06)				
x_t^h	=	- 0.27	$(r_t - E_t[\pi_{t+1}])$	-	0.23	$\hat{b}_{p,t}$	+	1.07	$E_t[x_{t+1}^h]$
		(0.17)			(0.07)			(0.06)	
x_t^h	=	- 0.16	$(r_t - E_t[\pi_{t+1}])$	-	0.20	$\hat{b}_{p,t}$	+		$E_t[x_{t+1}^h]$
		(0.14)			(0.05)				
x_t^e	=	- 0.13	$(r_t - E_t[\pi_{t+1}])$	-	0.16	$\hat{b}_{p,t}$	+	0.97	$E_t[x_{t+1}^e]$
		(0.07)			(0.03)			(0.05)	
x_t^e	=	- 0.15	$(r_t - E_t[\pi_{t+1}])$	-	0.16	$\hat{b}_{p,t}$	+		$E_t[x_{t+1}^e]$
		(0.07)			(0.03)				
x_t^c	=	- 0.13	$(r_t - E_t[\pi_{t+1}])$	-	0.18	$\hat{b}_{p,t}$	+	1.01	$E_t[x_{t+1}^c]$
		(0.11)			(0.04)			(0.03)	
x_t^c	=	- 0.11	$(r_t - E_t[\pi_{t+1}])$	-	0.17	$\hat{b}_{p,t}$	+		$E_t[x_{t+1}^c]$
		(0.08)			(0.03)				
x_t	=	- 0.44	$(r_t - E_t[\pi_{t+1}])$	-	0.13	\hat{b}_t	+	1.04	$E_t[x_{t+1}]$
		(0.21)			(0.03)			(0.03)	
x_t	=	- 0.17	$(r_t - E_t[\pi_{t+1}])$	-	0.10	\hat{b}_t	+		$E_t[x_{t+1}]$
		(0.14)			(0.03)				

Note: The first and second equations correspond to backward-looking specifications, while other equations correspond to the forward-looking specification. In addition, x_t^h is the HP-filtered real GDP, x_t^e is the HP-filtered personal consumption expenditure, x_t^c is the HP-filtered real non-durable consumption, $\hat{b}_{p,t}$ is the HP-filtered ratio of the public debt to GDP, and \hat{b}_t is the HP-filtered real debt. The generalized method of moments is used to estimate the forward-looking specification and the instrument variables include two-quarter and four-quarter lagged real interest rate, debt-to-GDP ratio, and output gap measures. The numbers in parenthesis are standard errors.

Figure 1: The Cross-Correlations between the Debt-GDP Ratio and the Output Gap

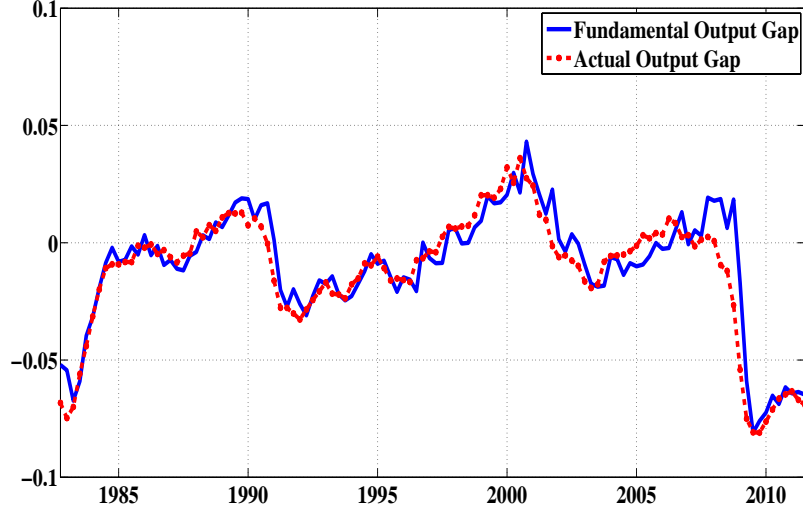


Note: The number at period i in this figure is the correlation of the current output gap at period t with the debt-GDP ratio at period $(t - i)$ for $i = -5, \dots, 5$. The output gap is defined as the percentage deviation of output from its potential GDP.

If the public debt has adverse effect on the aggregate demand, it is expected that coefficients of the real interest rate and the ratio of debt-GDP are positive in the backward-looking specification but they are both negative in the forward-looking specification. The estimation results from three different specifications uniformly imply that the aggregate demand falls as the ratio of the public debt to GDP rises. The estimates of the interest-elasticity (of the aggregate demand) that are implied from backward-looking regression equations are -0.15 and -0.12 respectively and its forward-looking version (the third line) is -0.42 as shown in Table 1. The estimates of the debt-elasticity (of the aggregate demand) that are implied from backward-looking regression equations are -0.20 and -0.21 respectively and its forward-looking version (the third line) is -0.27. In addition, the three estimates of the debt-elasticity are statistically significant at one percent significance level.

Furthermore, other measures of output gap are used to check the robustness of estimation results. For example, other measures include the HP-filtered real GDP, the HP-filtered personal consumption expenditure, and the HP-filtered real non-durable consumption. The interest-elasticities of aggregate consumption gaps that are derived from real personal consumption expenditure and real non-durable consumption appear to be smaller than those of output gap measures. But debt-elasticities turn out to be consistently and significantly negative across different output gap measures. Moreover, negative debt elasticities still show up in the forward-looking specification even when the debt-to-GDP ratio is replaced by the HP-filtered real debt, as shown in Table 1.

Figure 2: Fundamental and Actual Output Gaps



Note: The solid line depicts the output gap that is predicted by the empirical IS curve (fundamental output gap) and the dotted line represents the actual output gap. The x-axis corresponds to the sample period.

The key reason why the debt-to-GDP ratio has a negative coefficient in the empirical IS curve is that the current output gap is negatively correlated with future debt-GDP ratios. In order to understand the reason why these negative correlations are important, it is worthwhile to point out that the empirical IS curve is a forward-looking difference equation for the output gap, which means that the output gap can be written as an infinite sum of both future debt-GDP ratios and future anticipated real interest rates: $x_t = - \sum_{k=0}^{\infty} E_t[\psi_b(\hat{b}_{t+k} - x_{t+k}) + \psi_r(r_{t+k} - E_t[\pi_{t+k+1}])]$. Thus, a significantly positive estimate of ψ_b shows up when the current output gap has consistently and significantly negative correlations with future debt-GDP ratios.

Figure 1 reports cross-correlations between the debt-GDP ratio and the output gap. Specifically, the number at period i in this figure is the correlation of the current output gap at period t with the debt-GDP ratio at period $(t - i)$ for $i = -5, \dots, 5$. It is shown in this figure that the current output gap has negative correlations with future debt-GDP ratios. Thus, negative correlations of the current output gap with future debt-GDP ratios help understand the reason why the debt-elasticity has a negative estimate in the empirical IS curve.

Next, an alternative approach is used to see the performance of the empirical IS curve equation in the other context. Specifically, it is possible to use a VAR projection method, which is first proposed by Campbell and Shiller (1987) and used in Gali and Gertler (1999) and Sbordone (2002) for estimating the New Keynesian Phillips curve. A vector autoregression model is estimated and

used to obtain forecasts of the output gap, leading to estimates of coefficients of the empirical IS curve minimizing the sum of weighted squares of the difference between the actual output gap and the predicted output gap from the empirical IS curve. An unrestricted VAR model of three variables including the output gap, the anticipated real interest rate, and the ratio of debt to GDP is given by the following equation:

$$\tilde{z}_t = \Gamma_0 + \sum_{i=1}^s \Gamma_i \tilde{z}_{t-i} + e_t, \quad (2.3)$$

where the vector z_t is defined as $\tilde{z}_t = [x_t, r_t - E_t[\pi_{t+1}], \hat{b}_{p,t}]$, s is the lag length of the VAR model, and e_t represents a vector of the VAR residuals that are i.i.d. normal random variables with their means zero and variance-covariance matrix Ω . Moreover, the vector autoregression has a VAR(1) representation in the following form:

$$z_t = \Gamma z_{t-1} + \Phi e_t \quad (2.4)$$

where $z_t = [\tilde{z}_t, \tilde{z}_{t-1}, \dots, \tilde{z}_{t-s+1}]'$ and Γ is a matrix of coefficients. Having defined the coefficient vector as $\psi = [\psi_r, \psi_b]'$, the coefficient vector is estimated by solving the following minimization problem: $\min_{\psi} F(\psi, \hat{\Gamma})' \Xi(\hat{\Gamma}) F(\psi, \hat{\Gamma})$ where $\Xi(\hat{\Gamma})$ is a weighting matrix and $\hat{\Gamma}$ is the estimate of Γ . In addition, the vector $F(\psi, \hat{\Gamma})$ is defined as $F(\psi, \hat{\Gamma}) = S_x + (\psi_r S_r + \psi_b S_b)(I_{m \times m} - \hat{\Gamma})^{-1}$ where $m = 3s$, S_x is the selection vector for the output gap, S_r is the selection vector for the real interest rate, and S_b is the selection vector for the ratio of the public debt to GDP.

Having obtained the estimates of the empirical IS curve, it is now possible to construct the fundamental output gap by using the following equation:

$$x_t^f = -(\hat{\psi}_r S_r + \hat{\psi}_b S_b)(I_{m \times m} - \hat{\Gamma})^{-1} z_t \quad (2.5)$$

where x_t^f denotes the fundamental output gap at period t , $\hat{\psi}_r$ is the estimated elasticity of the interest rate, and $\hat{\psi}_b$ is the estimated elasticity of the debt-to-GDP ratio.

Turning the estimation of model parameters, the output gap (deviation of real GDP from potential GDP), the real anticipated interest rate, and the HP-filtered ratio of the public debt to GDP are used to estimate an unrestricted VAR model whose lag order is set to be $s = 4$. The minimum-distance estimators defined above lead to $\hat{\psi}_r = 0.28$ and $\hat{\psi}_b = 0.44$, given estimated parameters of the unrestricted VAR model. These estimates are thus consistent with those shown in Table 1. As discussed above, it is also possible to construct the time-series of output gap by using the empirical IS curve. Figure 2 demonstrates time-series of fundamental and actual output gaps. In this figure, the solid line corresponds to the fundamental output gap that is predicted by the empirical IS curve and the dotted line represents the actual output gap. The correlation between fundamental and actual output gaps is around 0.93. Thus, the fundamental output gap mimics the actual output gap remarkably well.

3 The Implication of the Empirical IS Curve for the Determinacy of the Equilibrium Dynamics

The main modelling emphasis of this paper is that the IS curve is the key equation for the transmission mechanism of the public debt. In order to see the reason why we care about the IS curve with the public debt, this section presents a brief discussion on its implication for the equilibrium dynamics.

A small DSGE model is used to see how the incorporation of the public debt into the IS curve affects the monetary and fiscal policy interaction. As discussed in the introduction, the IS curve is given by

$$x_t = \sigma_x E_t[x_{t+1}] - \sigma_r(r_t - E_t[\pi_{t+1}]) + \sigma_b \hat{b}_t. \quad (3.1)$$

The Phillips curve equation is

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa_x x_t \quad (3.2)$$

where β is the time discount factor of households and κ_x is the slope coefficient of the Phillips curve.¹ The central bank adopts the prototypical Taylor rule of the form:

$$r_t = \phi_\pi \pi_t + \phi_x x_t \quad (3.3)$$

where ϕ_π measures the responsiveness of the policy rate with respect to the aggregate inflation rate and ϕ_x measures these responsiveness of the policy rate with respect to the aggregate output gap. The government's one-period budget constraint is

$$\hat{b}_t = r_t + \beta^{-1}(\hat{b}_{t-1} - \pi_t) - \frac{1 - \beta}{\beta} s_t \quad (3.4)$$

where s_t is the log-deviation of the real primary surplus from its steady state. The fiscal rule is specified as follows:

$$s_t = \rho_s s_{t-1} + s_\pi \pi_t + s_x x_t + s_b \hat{b}_{t-1} + \epsilon_{s,t} \quad (3.5)$$

where s_π , s_x , and s_b are coefficients of inflation, output gap and lagged debt respectively, ρ_s is an AR(1) parameter of this surplus process, and $\epsilon_{s,t}$ is identically and independently distributed with mean zero and variance finite. In this section, the whole set of equilibrium conditions consists of equations (3.1), (3.2), (3.3), (3.4), and (3.5).

As shown above, the incorporation of the public debt into the IS curve requires the inclusion of both the government's one-period budget constraint and the fiscal policy rule into equilibrium conditions of the model. The interaction between monetary and fiscal policies should naturally take

¹Woodford (2003) provides a detailed discussion on the derivation of this Phillips curve from optimization problems of monopolistically competitive firms.

place if the public debt is included in the IS curve. The first topic of this section is the relation between the Taylor principle and the determinacy of the equilibrium path under non-Ricardian fiscal policy regime. It is well-known in the literature that the Taylor principle does not lead to a bounded unique equilibrium path if the government insists a non-Ricardian fiscal policy regime, as shown in Woodford (1995). An example of a non-Ricardian fiscal policy regime is that the real primary surplus follows an AR(1) process and its shocks are exogenously determined:

$$s_t = \rho_s s_{t-1} + \epsilon_{s,t}.$$

But, once the public debt is included in the IS curve, this well-known result does not hold uniformly even under a fiscal policy regime that is deemed to be non-Ricardian. If the coefficient of the public debt σ_b takes a negative value, the local uniqueness of the equilibrium path is restored with the Taylor rule even the non-Ricardian fiscal policy regime.²

In order to get a sense of how this result might come out, the whole set of equilibrium conditions can be collected into a system of linear difference equations. The resulting system of linear difference equations is summarized in the following matrix equation:

$$\begin{pmatrix} E_t[\pi_{t+1}] \\ E_t[x_{t+1}] \\ \hat{b}_t \\ E_t[s_{t+1}] \end{pmatrix} = \begin{pmatrix} A & \vdots & B \\ \cdots & \cdots & \cdots \\ C & \vdots & D \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \\ \hat{b}_{t-1} \\ s_t \end{pmatrix} \quad (3.6)$$

where 2×2 matrices A , B , C and D are defined as

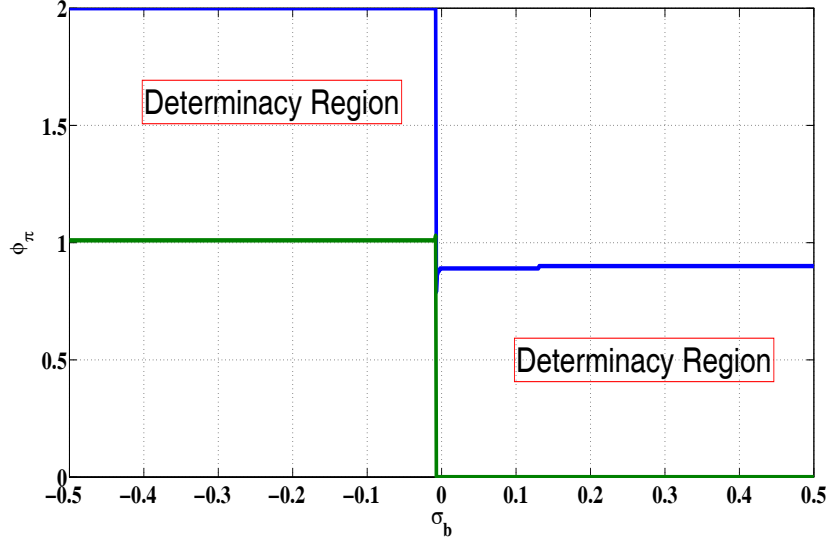
$$\begin{aligned} A &= \begin{pmatrix} \beta^{-1} & -\kappa_x \beta^{-1} \\ \sigma_k(\phi_\pi - \beta^{-1}) & \varphi_x - \sigma_b \phi_x \sigma_x^{-1} \end{pmatrix} & B &= \begin{pmatrix} 0 & 0 \\ -\sigma_b \beta^{-1} \sigma_x^{-1} & 0 \end{pmatrix} \\ C &= \begin{pmatrix} \phi_\pi - \beta^{-1} & \phi_x \\ 0 & 0 \end{pmatrix} & D &= \begin{pmatrix} \beta^{-1} & -(1-\beta)\beta^{-1} \\ 0 & \rho_s \end{pmatrix} \\ \varphi_x &= (1 + \sigma_r(\kappa_x \beta^{-1} + \phi_x))\sigma_x^{-1} & \sigma_k &= (\sigma_r - \sigma_b)\sigma_x^{-1} \end{aligned}$$

It should be noted that, if $\sigma_b = 0$, this system of linear difference equations becomes identical to that of the standard New Keynesian model with government's budget constraint, which is comparable to the one discussed in Woodford (1998). Specifically, the system becomes the one with a block diagonal matrix. In this case, the determinacy of equilibrium path requires that only one eigen-value of (2×2) matrix A should be greater than one in absolute value. Thus, maintaining the Taylor principle leads to an explosive path, rather than generating a unique stationary equilibrium path.

The interaction of spreads with the public debt ($\sigma_b < 0$) creates a new channel through which the public debt affects the aggregate IS curve. Thus, the block-diagonal system of linear difference

²Leeper (1991) develops concepts of active/passive monetary and fiscal policies based on their contribution in generating unique bounded equilibrium path. It is also possible to address the issues analyzed in this paper, based on those concepts of active and passive monetary and fiscal policies.

Figure 3: The Elasticity of the Public Debt and the Determinacy Region



Note: The AR(1) parameter of the exogenous primary surplus is $\rho_s = 0.95$ in order to produce this figure. In addition, σ_b denotes the elasticity of the public debt and ϕ_π is the inflation coefficient of the policy rule.

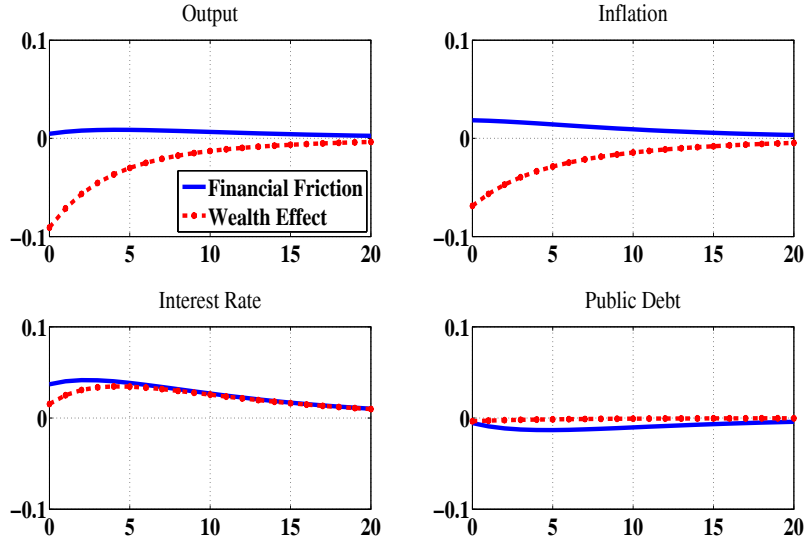
equations becomes broken down with the interaction between the public debt and interest-rate spreads. In this case, the IS curve equation implies that a rise in the public debt lowers the current aggregate demand, thus leading to falls in the aggregate inflation. Given this observation, the determinacy of equilibrium path comes with $\phi_\pi > 1$ (aggressive responses of the target rate with respect to the aggregate inflation).³

In order to see this result, suppose that there is an increase in the aggregate inflation and the central bank maintains the Taylor principle. Given this situation, the real interest cost of the public debt should increase in response to a rise in the aggregate inflation. The resulting increase in the public debt helps move down future inflation. The Taylor principle therefore turns out to be stabilizing.

Turning to the case of $\sigma_b > 0$, the IS curve equation implies that a rise in the public debt raises the current aggregate demand, thus leading to rises in the aggregate inflation. The determinacy of equilibrium path therefore comes with $0 < \phi_\pi < 1$ (mild responses of the interest-rate target with respect to the aggregate inflation). For the real interest cost of the public debt should fall in response to a rise in the aggregate inflation, thus leading to a decrease in the public debt. A fall in the real public debt then moves down future inflation and aggregate demand through its effect on the IS curve. Hence, mild responses of the target rate with respect to the aggregate inflation act

³Canzoneri, Cumby, and Diba (2011) contains a comprehensive survey on important issues in the monetary-fiscal interactions.

Figure 4: Dynamic Effects of Real Primary Surplus Shocks



Note: This figure shows impulse responses of aggregate variables when real budget surplus rises by one percent. The exogenous real primary surplus follows an AR(1) process while its persistence parameter is set to be 0.5.

as a stabilizing force for the aggregate inflation process.

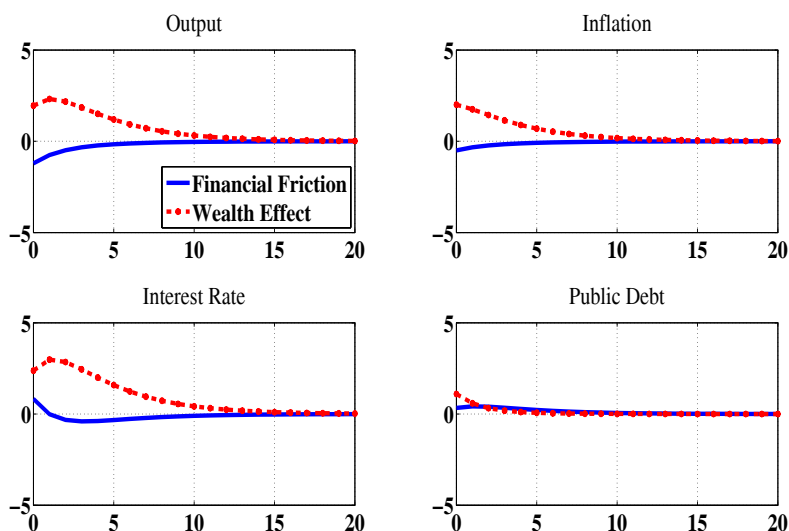
Figure 3 demonstrates pairs of values of the public debt’s elasticity and the coefficient of the aggregate inflation rate in the monetary policy rule that help produce a unique bounded equilibrium path, given a set of parameters. As shown in Figure 3, keeping the Taylor principle helps restore the uniqueness of local equilibrium path around the deterministic steady state when $\sigma_b < 0$. This result holds true even when the real primary surplus is exogenously determined. When $\sigma_b > 0$, however, keeping the Taylor principle is not helpful in obtaining the uniqueness of local equilibrium path around the deterministic steady state.

4 Business-Cycle Implications of the IS Curve with the Public Debt

It will be shown in this section that a change in the sign of the public debt’s coefficient (in the IS curve) affects qualitatively the way in which aggregate variables respond to changes in exogenous variables. In addition, the case of $\sigma_b > 0$ is represented as that of “wealth effect” and the case of $\sigma_b < 0$ corresponds to that of “financial friction,” which will be clarified later.

Figure 4 demonstrates dynamic responses of output and inflation with respect to an exogenous increase in the real primary surplus. The straight line corresponds to the impulse responses in the presence of financial frictions, while the dotted line corresponds to the case of “wealth effect”.

Figure 5: Dynamic Effects of Monetary Policy Shocks



Note: This figure shows impulse responses of aggregate variables when there is an exogenous rise in the policy rate. The exogenous monetary shock follows an AR(1) processes while its persistence parameter is set to be 0.5.

As shown in Figure 4, a rise in the real primary surplus moves down the aggregate inflation rate and output in the absence of financial frictions, whereas it raises both of them in the presence of financial frictions.

The public debt normally falls when the real primary surplus rises. But the impact of this decrease in the public debt on the aggregate demand differs depending on the sign of the elasticity of the public debt. For example, a reduction in the public debt raises the aggregate demand in the case of financial friction and thus increases the aggregate inflation as well. On the contrary, in the case of “wealth effect,” the aggregate demand drops down with a fall in the public debt, while the aggregate inflation also falls.

Figure 5 show dynamic impacts of monetary policy shocks. The straight line corresponds to the impulse responses in the presence of financial frictions, while the dotted line corresponds to the case of wealth effect. In this figure, inflation rate and output fall in the case of “financial friction”, whereas inflation and output rise in the case of “wealth effect”. In sum, when $\sigma_b < 0$, responses of inflation and output are qualitatively similar as those shown in the Ricardian regime with the Taylor principle. However, responses of inflation and output turn out to be qualitatively different as the sign of σ_b becomes positive. In this case, non-Ricardian fiscal regime needs to be set for the uniqueness of bounded equilibrium path.

The reason why responses of inflation and output are different in the two cases can be explained

as follows. Although a rise in the nominal interest rate leads to an increase in the interest cost in both of the two cases, the resulting increase in the real public debt has different impacts on the aggregate demand. In particular, the resulting increase in the real public debt raises the aggregate demand in the case of “wealth effect.” Thus, inflation rises substantially, should the real public debt be inflated away. But in the case of “financial friction,” the increase in the real public debt due to the rise in the nominal interest rate decreases the aggregate demand. Hence, the inflation rate falls as well. Moreover, the aggressive monetary policy rule reduces the real interest rate. As a result, subsequent decreases in the aggregate demand helps curtail the initial rise of the nominal interest rate.

5 Fiscal Policy Regimes and the Forward Guidance at the Zero Lower Bound

The specification of the IS curve plays an important role in the power of the forward guidance at the zero lower bound on the short-term nominal rate of interest, as will be shown in this section. In addition, even without any changes in the specification of the IS curve, only switches of anticipated future fiscal regimes can have significant impact on the effectiveness of the forward guidance.

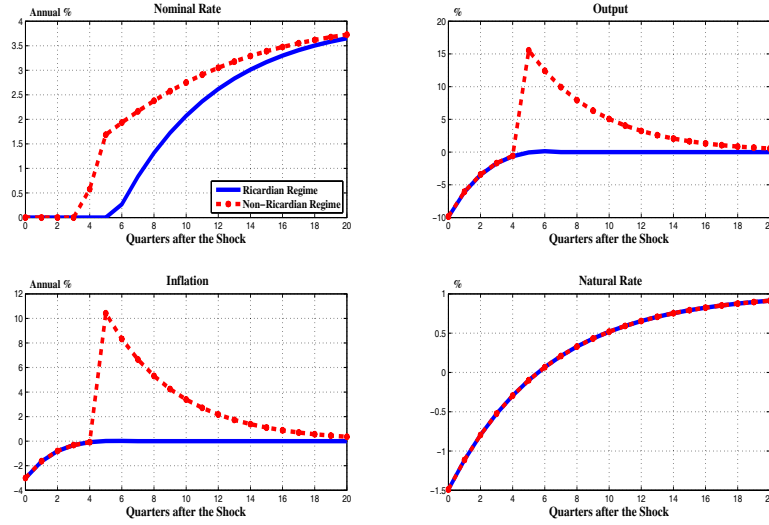
A prototypical New Keynesian model is used to see the impact of agents’ expectations about future fiscal regime in the effectiveness of the forward guidance that should work at the zero lower bound on the nominal interest rate. In order to do this, let’s suppose that an unanticipated exogenous shock at time $t = 0$ causes the natural real interest rate r_t^n to drop below zero and to remain negative until period $t = T$ and then return to positive values from period $T + 1$ onward, following Jung, Teranish, and Watanabe (2006) and Levin, Lopez-Salido, Nelson, and Yun (2010). With the perfect foresight assumption, the full exogenous path of the natural rate is known to agents after the shock in period 0 has moved down the natural rate. In this case, the short-term nominal policy rate may remain at its zero lower bound even after the natural rate resumes positive values.⁴

By iterating backward the IS and Phillips curve equations from the terminal point of the zero lower bound, the pair of inflation and the output gap under the zero lower bound can be expressed as a function of the expected values of these variables at period $T + 1$ as follows:

$$\begin{pmatrix} \pi_{T-i} \\ x_{T-i} \end{pmatrix} = A^{i+1} \begin{pmatrix} \pi_{T+1} \\ x_{T+1} \end{pmatrix} + \sigma \sum_{s=0}^i A^s \begin{pmatrix} \kappa_x \\ 1 \end{pmatrix} r_{t-i+s}^n. \quad (5.1)$$

⁴In Levin, Lopez-Salido, Nelson, and Yun (2010, LLNY hereafter), the natural rate shock follows an AR(1) process with first-order autocorrelation coefficient $\rho = 0.75$ for the “Great Moderation”-style shock and $\rho = 0.85$ for the “Great Recession”-style shock. In the simulation, the natural rate follows the specification of the “Great Recession”-style shock. In addition, other parameter values are taken from LLNY (2010): $\kappa_x = 0.035$, $\sigma = 1$, and $\beta = 0.99$. Those values will be used throughout the rest of this paper.

Figure 6: Anticipated Future Fiscal Policy Regimes and the Forward Guidance at the ZLB

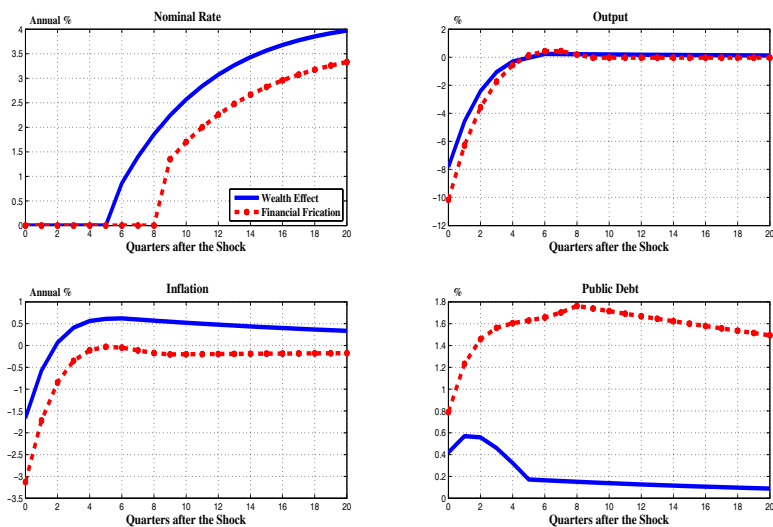


Note: The AR(1) parameter of the natural rate shock is $\rho = 0.85$, while the initial shock to the natural rate is $\epsilon_0^n = -0.025$.

The main point is that the forward guidance vector $[x_{T+1}, \pi_{T+1}]$ serves as a terminal condition in period $T + 1$ that pins down the equilibrium outcomes of inflation and output gap for the preceding periods. Given this characterization of the forward guidance, it is clear that once agents begin to form inflationary expectations because of their expectations about future fiscal regimes, those changes in their expectations should be reflected in the forward guidance vector $[x_{T+1}, \pi_{T+1}]$. In other words, the inflationary expectations may raise the power of the forward guidance by raising π_{T+1} . In particular, others being equal, once the non-Ricardian fiscal policy regime happens to take place, the increased power of the forward guidance may yield a prescription of a shorter duration of the zero lower bound for the same magnitude of the shock to the natural rate of interest rate, as shown in Figure 6.

Figure 6 shows the impact of anticipated future fiscal policy regimes on the forward guidance at the ZLB. When agents expect a non-Ricardian fiscal policy regime, the inflation and output gap rise rapidly after the ZLB period. However, the inflation and output gap become zero soon after the economy gets out of the ZLB. The discrepancy in the vectors of the forward guidance (that can be implied by the panels) is derived only from the difference between anticipated fiscal policy regimes that might prevail after the ZLB period. Furthermore, it should be mentioned that coefficients of inflation and output are set to be $\phi_\pi = 1.5$ and $\phi_x = 0.5$ (for the monetary policy rule) in the case of Ricardian fiscal policy regime and coefficients of inflation and output are set to be $\phi_\pi = 0.2$

Figure 7: The IS Curve with the Public Debt and the Forward Guidance at the ZLB



Note: The AR(1) parameter of the natural rate shock is $\rho = 0.85$, while the initial shock to the natural rate is $\epsilon_0^n = -0.025$. In addition, $\sigma_b = 0.05$ in the case of “wealth effect” and $\sigma_b = -0.05$ in the case of “financial friction effect.”

and $\phi_x = 0$ in the case of non-Ricardian fiscal policy regime in order to have a unique bounded equilibrium path.

Figure 7 shows the implication of the IS curve with the public debt on the forward guidance. The “wealth effect” case means $\sigma_b = 0.05$ and the “financial friction” case means $\sigma_b = -0.05$. The impact of the rise in the public debt on the effectiveness of forward guidance might differ, depending on whether or not the IS curve reflects the wealth/liquidity effect of the public debt or its effect on the financial friction. Specifically, when financial friction increases with the rise of the public debt, increases in the public debt during the ZLB period move down the effectiveness of forward guidance but the wealth effect of the public debt strengthens the forward guidance of the zero lower bound. The duration of the ZLB therefore lengthens in the case of $\sigma_b = -0.05$. For example, as shown in Figure 7, the number of the ZLB period is 8 quarters in the case of “financial friction” while it is 5 quarters in the case of “wealth effect.” As is discussed in previous section, the “wealth effect” case corresponds to the non-Ricardian regime, so that coefficients of inflation and output are set to be $\phi_\pi = 0.2$ and $\phi_x = 0$ as a way of having a unique bounded equilibrium path. The “financial friction” case does not necessarily need to turn away from the Taylor principle. In this case, thus, coefficients of inflation and output are set to be $\phi_\pi = 1.5$ and $\phi_x = 0.5$ for the monetary policy rule.

6 Market Friction of Government Securities and Sovereign-Risk Based Channel

In this section, a small DSGE model with market friction of government securities and the default risk of government debt is used to obtain a theoretic IS curve that is identical to the specification of the empirical IS curve. In the model of this section, primary and secondary markets exist for government securities. Market friction for government securities arise because of the presence of entry barrier for participants of the primary market who can act as primary dealers. The participants of the secondary market are retail banks at which households make their deposits and obtain loans. Because of this entry barrier, monopoly profits from dealing with government securities can arise in the presence of sovereign default risk. Given this market structure, sovereign default risk could be transferred to the participants of the secondary market.

In order to see this issue, let's suppose that the government promises $R_{p,t}$ as the gross rate of return for holding its short-term securities between periods t and $t + 1$. However, sovereign default risk exists at period $t + 1$, while δ_{t+1} is the default rate at period $t + 1$. In this case, the gross rate of return at the primary market is $R_{p,t}E_t[(1 - \delta_{t+1})]$ and the gross rate of return at the secondary market is $R_{p,t}$. Since the unit profit of primary dealers from dealing with short-term government securities is $R_{p,t}E_t[\delta_{t+1}]$, the segmented structure of the securities markets brings about positive profits in the presence sovereign default risk.⁵

Furthermore, when we set $E_t[Q_{t,t+1}] = R_{p,t}^{-1}$, the optimization condition of households for bond-holdings implies $\beta R_{p,t}E_t[\lambda_t/(\lambda_{t+1}\Pi_{t+1})] = 1$. In addition, the central bank sets its target on the short-term interest rate of primary dealers: $R_{p,t}E_t[(1 - \delta_{t+1})] = \Pi_t^{\phi_\pi} Y_t^{\phi_x}$. The government's one-period budget constraint at period t is also given by $b_t = R_{p,t}E_t[(1 - \delta_{t+1})](b_{t-1}/\Pi_t - s_t)$. As a result, the log-linearized version of the Euler equation is

$$x_t = E_t[x_{t+1}] - \sigma(r_t - E_t[\pi_{t+1}]) - \sigma E_t[\hat{\delta}_{t+1}] \quad (6.1)$$

where r_t denotes the policy rate of the central bank, σ is the elasticity of intertemporal substitution, $\hat{\lambda}_t = \sigma^{-1}x_t$ is used to derive this IS curve, and $\hat{\delta}_t (= -\log(1 - \delta_t)/(1 - \delta))$ denotes the log-deviation of the default rate of government securities.

In the era of persistently high public debt, agents expect that future default rates are positively associated with the ratio of the public debt to GDP:

$$E_t^*[\hat{q}_{t+1}] = -\psi_q(\hat{b}_t - x_t) \quad (6.2)$$

⁵Fiscal limit can be endogenously determined if setting tax rates is constrained by the Laffer curve. Bi (2010) and Davig Leeper and Walker (2010) discuss inflationary consequences of fiscal limit and sovereign default risk.

Table 2: Log-Linearized Equilibrium Conditions

$$\begin{aligned}
 x_t &= \sigma_x E_t[x_{t+1}] - \sigma_r(r_t - E_t[\pi_{t+1}]) + \sigma_b \hat{b}_t \\
 \pi_t &= \beta E_t[\pi_{t+1}] + \kappa_x x_t \\
 r_t &= \phi_\pi \pi_t + \phi_x x_t \\
 \hat{b}_t &= r_t + \beta^{-1}(\hat{b}_{t-1} - \pi_t) - \beta^{-1}(1 - \beta)s_t
 \end{aligned}$$

Note: The first equation is the IS curve, the second equation corresponds to the Phillips curve equation, the third equation is the Taylor rule, and the fourth equation is the log-linearized government's budget constraint.

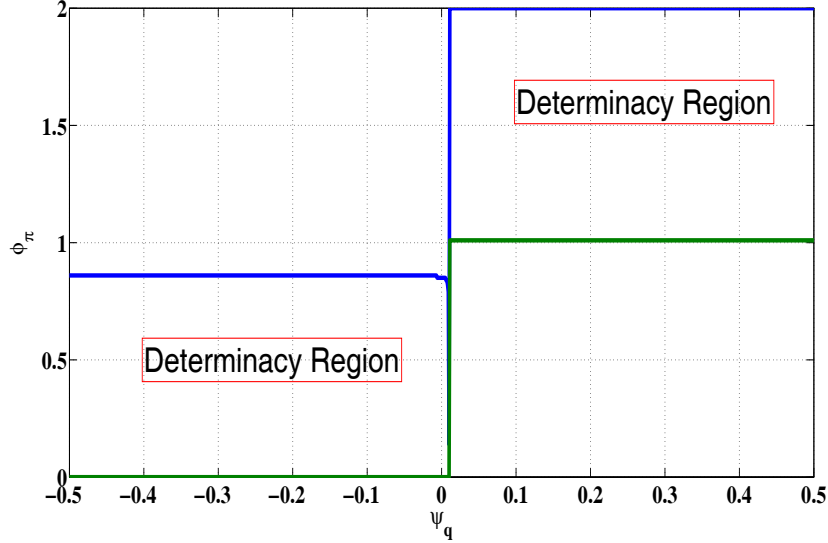
where the coefficient $\psi_q (> 0)$ is positive. As an empirical evidence for this relation, the recent empirical findings of Acharya, Drechsler, and Schnabl (2011) on the relation between sovereign and bank CDS during the period of 2007-2010 can be summarized as follows:

- Pre-Bailout Period: This period covers the start of the financial crisis in January 2007 until the bankruptcy of Lehman Brothers. Across all Western economies, there was a large, sustained rise in bank CDS as the financial crisis develops. However, sovereign CDS spreads remains very low. This evidence is consistent with a significant increase in the default risk of the banking sector with little effect on sovereigns in the pre-bailout period.
- Bailout Period: This period covers the bank bailouts starting with the announcement of a bailout in Ireland in late September 2008 and ending with a bailout in Sweden in late October 2008. During this one-month period, there was a significant decline in bank CDS across all countries and a corresponding increase in sovereign CDS. This evidence suggests that bank bailouts produced a transfer of default risk from the banking sector to the sovereign.
- Post-Bailout Period: This period covers the period after the bank bailouts and until 2010. both sovereign and bank CDS increased during this period and that the increase was larger for countries with significant public debt ratios. This evidence suggests that the banks and sovereigns share the default risk after the announcement of banks bailouts and that the risk is increasing in the relative size of countries' public debt.

Combining equation (6.1) with equation (6.2), the log-linearized IS curve turns out to be

$$x_t = \sigma_x E_t[x_{t+1}] - \sigma_r(r_t - E_t[\pi_{t+1}]) + \sigma_b \hat{b}_t \quad (6.3)$$

Figure 8: The Elasticity of the Public Debt and the Determinacy Region



Note: The AR(1) parameter of the exogenous primary surplus is $\rho_s = 0.95$ in order to produce this figure. In addition, σ_b denotes the elasticity of the public debt and ϕ_π is the inflation coefficient of the policy rule. The output coefficient is set to be 0.5.

where coefficients of the IS curve σ_x , σ_r , and σ_b are now defined as

$$\sigma_x = \frac{1}{1 - \sigma\psi_q} \quad \sigma_r = \frac{\sigma}{1 - \sigma\psi_q} \quad \sigma_b = -\frac{\sigma\psi_q}{1 - \sigma\psi_q}.$$

In this case, the sign of σ_b is negative so that this model belongs to the case of “financial friction” explained in the previous section. Table 2 also includes a set of log-linearized equations that can be used to compute the equilibrium dynamics of a small DSGE model.

Figure 8 demonstrates pairs of values of the public debt’s elasticity for the risk premium and the coefficient of the aggregate inflation rate in the monetary policy rule that guarantee a unique equilibrium path. As shown in Figure 8, keeping the Taylor principle helps restore the uniqueness of local equilibrium path around the deterministic steady state when ψ_q takes a positive value (not too close to zero) when the real primary surplus is exogenously determined. However, when $\psi_q < 0$, keeping the Taylor principle is not helpful in obtaining the uniqueness of local equilibrium path around the deterministic steady state.

7 Time-Varying Risk Preferences and Convenience-Yield Based Channel

The aim of this section is to present an equilibrium model in which the specification of the estimated empirical IS curve discussed above can be interpreted as an approximation to the model’s IS curve

under plausible parameter values (up to the first-order approximation). Moreover, the model of this section does not necessarily need to introduce an explicit specification of sovereign default risk premium, which is in contrast with the model of the previous section.

Investors might want to prefer a short maturity of government securities than a long maturity especially when they perceive a substantial amount of sovereign default risk. This time-varying attitude toward maturities of government securities can be formulated in the model with convenience yield of government securities by assuming that the marginal utility benefits of holding short-term government securities rise as the total size of the public debt increases. In order to simplify the analysis, the time-varying coefficient for the benefit of short-term government securities rises when the aggregate size of the public debt increases relative to the aggregate nominal output. Hence, the preferences of households at period 0 are represented by the following equation:

$$\sum_{t=0}^{\infty} \beta^t E_0 [u(c_t(h), 1 - n_t(h)) + e(\frac{B_t}{P_t Y_t}) v(\frac{B_{s,t}(h)}{P_t})] \quad (7.1)$$

where $B_{s,t}(h)$ is the household h 's nominal holding of short-term government securities, B_t is the aggregate public debt in nominal term, and $v(B_{s,t}(h)/P_t)$ denotes the utility level at period t from short-term government securities (that is concave and continuously twice differentiable in its argument). The function $e(B_t/(P_t Y_t))$ is a monotonically increasing function and thus drives the impact of time-varying aversion to longer-term securities as the public debt rises.

In order to defend the inclusion of the short-term government debt in the utility function of an individual household, it would be worthwhile to mention the work of Krishnamurthy and Vissing-Jorgensen (2010) who report three reasons for holding Treasuries: 1) the high liquidity of Treasuries compared to corporate bonds; 2) neutrality, which may motivate official institutions such as U.S. Federal Reserve banks, state and local governments, and foreign central banks to hold Treasuries to avoid favoring any non-governmental borrower over another; and 3) safety of Treasuries (their widespread reputation as the lowest-risk interest-bearing asset).

Furthermore, government issues two different types of securities: One is one-period discount bond whose face value is one dollar and the other is a nominal consol whose nominal coupon is one at the initial period and then decays geometrically at a rate of δ over time. The dollar price at period t of this nominal consol in its initial period is $P_{c,t} = 1 + \delta E_t[Q_{t,t+1} P_{c,t+1}]$, where $Q_{t,t+1}$ is the nominal stochastic discount factor that is used to compute the dollar value at period t of one dollar at period $t + 1$. The price at period t of the nominal consol issued at period $t - k$ is $\delta^k P_{c,t}$ for $k = 1, 2, \dots, \infty$. The one-period holding return of a nominal consol is the same regardless of the time period when it is issued: $R_{c,t+1} = (1 + \delta P_{c,t+1})/P_{c,t}$. The budget constraint at period t of

household h is then given by

$$c_t(h) + \frac{B_{s,t}(h)}{P_t R_t} + \frac{P_{c,t} B_{c,t}(h)}{P_t} \leq w_t n_t(h) + \frac{B_{s,t-1}(h)}{P_t} + \frac{(1 + \delta P_{c,t}) \sum_{k=1}^{\infty} \delta^{k-1} B_{c,t-k}(h)}{P_t} + \Phi_t \quad (7.2)$$

where w_t is the real wage at period t and Φ_t is the real dividend income at period t . Hence, the household's utility optimization condition for holding any nominal consol is

$$\beta E_t \left[\frac{\lambda_{t+1} R_{c,t+1}}{\lambda_t \Pi_{t+1}} \right] = 1 \quad (7.3)$$

where λ_t is the marginal utility of consumption at period t and $\Pi_{t+1} (= P_{t+1}/P_t)$ is the ratio of the price level at period $t+1$ to the price level at period t . The optimization condition for holding short-term government securities between periods t and $t+1$ is

$$\lambda_t = R_t \left(e \left(\frac{b_t}{y_t} \right) v' (b_{s,t}) + \beta E_t \left[\frac{\lambda_{t+1}}{\Pi_{t+1}} \right] \right) \quad (7.4)$$

where $b_{s,t} (= B_{s,t}/P_t)$ is the real value of short-term government bonds held by household h and b_t is the real total debt at period t (held by the public).

Moreover, the instantaneous government's budget constraint can be written as

$$\frac{B_{s,t}}{R_t} + P_{c,t} B_{c,t} = C_{p,t} + B_{s,t-1} - P_t s_t \quad (7.5)$$

where $C_{p,t}$ is the total nominal coupon payments at period t and $B_{c,t}$ is the face value of the nominal consol issued at period t . The geometric decay of coupons over time leads to a simple evolution equation for $C_{p,t}$: $C_{p,t} = B_{c,t} + \delta C_{p,t-1}$. The aggregate nominal public debt at period t (evaluated in terms of the market value) is defined as $B_t = B_{s,t}/R_t + P_{c,t} C_{p,t}$.

7.1 Implication for the relation between the public debt and maturity

The empirical relation between the size of the public debt and the maturity structure of government securities exhibits a non-linear relation. For example, Missale and Blanchard (1994) found a strong inverse relation between maturity and the debt-to-GDP ratio for countries which reached debt-to-GDP ratios approaching or exceeding 100 percent.

An explanation about this observation is that high inflation is associated with higher inflation uncertainty, leading to higher risk premium on long-term nominal debt and thus governments to stop issuing long-term debt. The explanation explored by Missale and Blanchard is that the government would choose a shorter maturity in order to make its non-inflation pledge especially when the government might have a strong incentive to inflate away its nominal debt.

The model of this section can be used to bring about an independent explanation for the observed inverse relation between the size of the public debt and the maturity structure of government securities. The risk aversion of investors for government securities is associated with the level of

the public debt relative to GDP. In this case, a higher debt-to-GDP ratio leads to a higher risk premium on long-term nominal debt and thus governments to stop issuing long-term debt.

The risk premium of long-term government securities over short-term government securities is defined as the difference between the expected one-period holding return of long-term government securities and the risk-free rate of short-term government securities. By log-linearizing equations (7.3) and (7.4) and then combining the resulting two equations, it can be shown that the risk premium of a long-term bond still exists even in the log-linear version of the model:

$$E_t[r_{c,t+1}] - r_t = \frac{1 - R\beta}{R\beta}(r_t + \sigma^{-1}x_t + \epsilon_b(\hat{b}_t - x_t) - \epsilon_s\hat{b}_{s,t}) \quad (7.6)$$

where $\epsilon_b(= e'(x)x/e(x))$ and $\epsilon_s(= -v''(x)x/v'(x))$ are the elasticity of function $e(x)$ with respect to x and the elasticity of marginal utility of government bonds. The presence of the convenience yield of short-term government securities drives the wedge between the real gross interest rate and the inverse of time discount factor: $\beta R < 1$. Hence, to the extent that the convenience yield of short-term government securities exists, the risk premium of long-term government securities rises as the ratio of the public debt to GDP rises even in the log-linearly approximated model.

7.2 Implication for monetary-fiscal policy interactions

Table 3 summarizes the whole set of log-linearized equilibrium conditions. The first line in the table corresponds to the log-linearized Euler equation that can be interpreted as the IS curve of this model. The ratio of the public debt-to-GDP is included in the IS curve, while its coefficient is negative. Hence, the modified version of the convenience model helps explain the reason why the ratio of the public debt-to-GDP is significantly included in the empirical IS curve along with its coefficient negative.

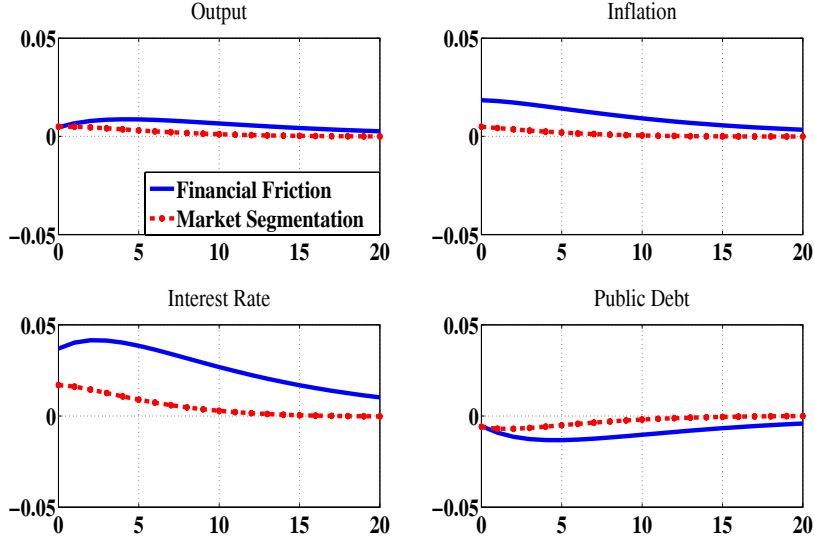
Turning to the calibration of parameter values, Krishnamurthy and Vissing-Jorgensen (2010) report that the average convenience yield of long-term Treasuries over the period of 1926-2008 is 72 basis points of which 46 basis points is driven by the liquidity of Treasuries and 26 basis points by the safety of Treasuries. Moreover, since the pecuniary return and the convenience yield together create the motivation of holding Treasuries, $R + c = \beta^{-1}$ should hold at the deterministic steady state where c denotes the steady-state convenience yield. Under the assumption that the average convenience yield of short-term Treasuries is similar with this reported value, $c = 0.0072$. Laubach (2009) reported that a percentage point increase in the projected deficit to GDP ratio raises the five-year-ahead 10-year forward rate by 20 to 29 basis points: a typical estimate is about 22 basis points. If a percentage point increase in the debt to GDP ratio creates a similar magnitude of the estimate and the short-term interest rate is not significantly affected by this increase in the projected deficit to GDP ratio, it would be alright to set $\epsilon_b(1 - R\beta)/(R\beta) = 0.29$. Thus, given

Table 3: Log-Linearized Equilibrium Conditions

$$\begin{aligned}
 x_t &= \sigma_x E_t[x_{t+1}] - \sigma_r(r_t - E_t[\pi_{t+1}]) - \sigma_p(E_t[\pi_{t+1}] + \epsilon_b(\hat{b}_t - x_t) - \epsilon_s \hat{b}_{s,t}) \\
 \hat{b}_t &= \omega_s(b_{s,t} - r_t) + (1 - \omega_s)(\hat{c}_{p,t} + \hat{P}_{c,t}) \\
 \hat{b}_{s,t} &= r_t + R(\hat{b}_{s,t-1} - \pi_t) - \frac{(1-\delta)(1-\omega_s)}{\omega_s}(\hat{P}_{c,t} + \hat{b}_{c,t}) + \frac{(1-\beta\delta)(1-\omega_s)}{\omega_s}\hat{c}_{p,t} - \xi s_t \\
 r_{c,t+1} &= \beta\delta(\hat{P}_{c,t+1} - \hat{P}_{c,t}) \\
 \hat{c}_{p,t} &= (1 - \delta)\hat{b}_{c,t} + \delta(\hat{c}_{p,t-1} - \pi_t) \\
 E_t[r_{c,t+1}] &= r_t + \frac{1-R\beta}{R\beta}(r_t + \sigma^{-1}x_t + \epsilon_b(\hat{b}_t - x_t) - \epsilon_s \hat{b}_{s,t}) \\
 \pi_t &= \beta E_t[\pi_{t+1}] + \kappa_x x_t \\
 r_t &= \phi_\pi \pi_t + \phi_x x_t \\
 \hat{b}_{c,t} &= \phi_b \hat{b}_t
 \end{aligned}$$

Note: The coefficients σ_x , σ_r , σ_p , and γ are defined as $\sigma_x = R\beta$, $\sigma_r = \sigma R\beta$, $\sigma_p = (1 - R\beta)\sigma$, $\xi = (1/\omega_s)(s/b)$, while R is the steady-state gross interest rate and $\omega_s (= b_s/(b_s + RP_c c_p))$ is the share of short-term securities in the aggregate total debt.

Figure 9: Dynamic Effects of Real Primary Surplus Shocks



Note: This figure shows impulse responses of aggregate variables when real budget surplus rises by one percent. The exogenous real primary surplus follows an AR(1) processes while its persistence parameter is set to be 0.5.

that $\epsilon_b = 0.29(1 - \beta c)/(\beta c)$, $\beta = 0.99$, and $c = 0.0072$ are used to compute ϵ_b . Moreover, following Greenwood and Vayanos (2010), the average-maturity of Treasuries is 4.67 years and the share of long-term Treasuries is 0.18 at the deterministic steady-state equilibrium. Hence the maturity parameter is set to be $\delta = 0.78$ and the share of short-term debt is $\omega_s = 0.5$.

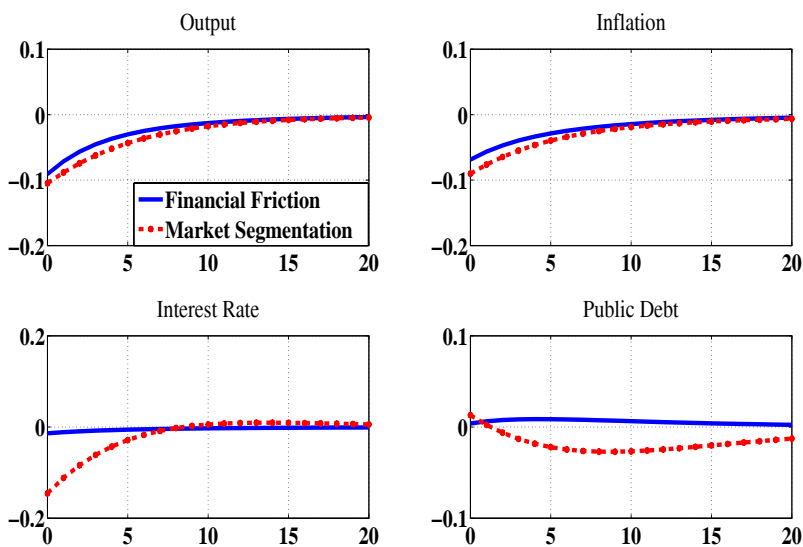
In particular, it is notable that $\sigma_p = \sigma\beta c$. Hence, when c is sufficiently small, the value of σ_p becomes very close to zero, leading to essentially zero coefficient of $\hat{b}_{s,t}$ if the size of ϵ_s is relatively small. Given these parameter values, it might be then plausible to argue that the IS curve equation in the first line of Table 3 can be approximated as follows:

$$x_t \approx \sigma_x E_t[x_{t+1}] - \sigma_r(r_t - E_t[\pi_{t+1}]) - \tilde{\sigma}_b(\hat{b}_t - x_t)$$

where the coefficient $\tilde{\sigma}_b$ is defined as $\tilde{\sigma}_b = 0.29\sigma(1 - \beta c)$.

Figure 9 shows the impulse responses of aggregate variables with respect to an exogenous rise in the real primary surplus. The straight line corresponds to the reduced form model of financial frictions, while the dotted line corresponds to the impulse responses in the convenience yield model with financial frictions. As shown in Figure 9, an increase in the real primary surplus raises both of inflation and output in the convenience yield model with the presence of financial frictions. The reason behind this result can be explained as follows. Households prefer short-term government bonds less than before as the public debt goes down. This feature in turn raises the required level of the short-term nominal interest rate to hold short-term government securities. Hence, the risk-

Figure 10: Dynamic Effects of Monetary Policy Shocks



Note: This figure shows impulse responses of aggregate variables with respect to an exogenous increase in the policy rate. The exogenous shock follows an AR(1) processes while its persistence parameter is set to be 0.5.

premium of long-term government securities drops as the public debt falls. The reduction of this risk premium helps boost up the aggregate demand. The increases in the aggregate demand also raises the inflation rate.

Figure 10 shows dynamic impacts of monetary policy shocks. The straight line corresponds to the reduced form model of financial frictions, while the dotted line corresponds to the impulse responses in the convenience yield model with financial frictions. As shown in this figure, the inflation rate and output fall in the convenience yield model with the “financial friction” effect. In this case, the increases in the short-term interest rate reduces the aggregate demand through the IS curve. The reduction of the aggregate demand translates into a fall in the aggregate inflation rate through the Phillips curve.

8 Market-Segmentation Based Channel

The aim of this section is to present a small DSGE model in which the dynamic responses of aggregate variables with respect to changes in exogenous variables are similar to those shown in the case of “financial friction,” while the model’s IS curve is not identical to those used in the empirical part of the paper.

In the model of this section, there are two different types of financial institutions. Commercial banks take deposits from households to hold short-term government bonds and make loans to other

financial institutions. Investment banks hold only long-term government securities in the asset side of their balance sheets. Because of this assumption for commercial and investment banks, investors are segmented between markets of short-term and long-term government securities. Since net-worths of investment banks are not enough for their financial investments, part of their investment funds must be borrowed from commercial banks. While multiplicative idiosyncratic shocks affect realized returns of investment banks, there is ex-post information asymmetry (between borrowers and lenders) regarding realized returns from holding long-term government securities, following Bernanke, Gertler and Gilchrist (1999). It is also assumed that lenders must pay audition fee to obtain the precise information about the realized return of an investment bank.

An investment bank's manager begins with his/her net worth N_t and borrows $M_{c,t} = P_{c,t}K_{c,t} - N_{c,t}$ where $P_{c,t}$ is the nominal price of long-term government securities and $K_{c,t}$ is the target purchase of long-term government securities. The realized profit of the investment bank can be written as $\omega R_{c,t+1}P_{c,t}K_{c,t} - Z_{c,t+1}M_{c,t}$, where $Z_{c,t+1}$ is the contract interest rate and ω is an i.i.d. random variable that follows a log-normal distribution $\log \omega \sim \mathcal{N}(-\sigma_\omega^2/2, \sigma_\omega^2)$. Hence, the threshold value of ω for a non-negative profit under this debt contract is $\bar{\omega}_{t+1}R_{c,t+1}P_{c,t}K_{c,t} = Z_{c,t+1}M_{c,t}$. In addition, the participation constraint of lenders can be written as

$$\int_{\bar{\omega}_{t+1}}^{\infty} \bar{\omega} R_{c,t+1} P_{c,t} K_{c,t} \phi(\omega) d\omega + (1 - \mu) \int_0^{\bar{\omega}_{t+1}} \omega R_{c,t+1} P_{c,t} K_{c,t} \phi(\omega) d\omega = R_t (P_{c,t} K_{c,t} - N_{c,t}).$$

As a result, the debt contract problem for investment and commercial banks can be written as

$$\max_{K_{c,t}, \bar{\omega}_{t+1}} (1 - \Gamma(\bar{\omega}_{t+1})) R_{c,t+1} P_{c,t} K_{c,t} \quad (8.1)$$

subject to

$$(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})) R_{c,t+1} P_{c,t} K_{c,t} = R_t (P_{c,t} K_{c,t} - N_t) \quad (8.2)$$

where $\Gamma(\bar{\omega}_{t+1})$ and $G(\bar{\omega}_{t+1})$ are defined as

$$\Gamma(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega \phi(\omega) d\omega + \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} \phi(\omega) d\omega \quad G(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega \phi(\omega) d\omega.$$

The optimization conditions of this debt contract imply that the aggregate investment on long-term government securities is determined by the following equation:

$$\frac{P_{c,t} K_{c,t}}{N_{c,t}} = \Psi\left(\frac{R_{c,t+1}}{R_t}\right) \quad (8.3)$$

where function $\Psi(x)$ is increasing in its argument.⁶ Furthermore, the net-worth of the investment bank's manager evolves over time according to the following equation:

$$N_{c,t} = (1 - \gamma) \{R_{c,t} P_{c,t-1} K_{c,t-1} - \mu G(\bar{\omega}_t) R_{c,t} P_{c,t-1} K_{c,t-1} - R_{t-1} (P_{c,t-1} K_{c,t-1} - N_{c,t})\} + \Omega P_t Y_t \quad (8.4)$$

⁶Bernanke, Gertler, and Gilchrist (1999) contains a detailed discussion on how to derive the function $\Psi(x)$ from the debt contract problem specified in equations (8.1) and (8.2).

Table 4: Log-Linearized Equilibrium Conditions

$$\begin{aligned}
x_t &= E_t[x_{t+1}] - \sigma E_t[r_{c,t+1} - \pi_{t+1}] \\
n_{c,t} &= \kappa_n(n_{c,t-1} + r_t - \pi_t + \epsilon_n(r_{c,t} - r_{t-1})) + (1 - \kappa_n)x_t \\
E_t[r_{c,t+1}] &= r_t + \epsilon_\psi^{-1}(\hat{P}_{c,t} + \hat{b}_{c,t} - n_{c,t}) \\
r_{c,t+1} &= \delta\beta(\hat{P}_{c,t+1} - P_{c,t}) \\
\hat{c}_{p,t} &= (1 - \delta)\hat{b}_{c,t} + \delta(\hat{c}_{p,t-1} - \pi_t) \\
\hat{b}_{s,t} &= r_t + R(\hat{b}_{s,t-1} - \pi_t) - \frac{(1-\delta)(1-\omega_s)}{\omega_s}(\hat{P}_{c,t} + \hat{b}_{c,t}) + \frac{(1-\beta\delta)(1-\omega_s)}{\omega_s}\hat{c}_{p,t} - \xi s_t \\
\hat{b}_t &= \omega_s(b_{s,t} - r_t) + (1 - \omega_s)(\hat{P}_t + \hat{c}_{p,t}) \\
\pi_t &= \beta E_t[\pi_{t+1}] + \kappa_x x_t \\
r_t &= \phi_\pi \pi_t + \phi_x x_t \\
\hat{b}_{c,t} &= \phi_b \hat{b}_t
\end{aligned}$$

Note: In this table, $n_{c,t}$ is the log-deviation of the real net-worth of investment banks, $\epsilon_\psi (= \Psi'(x)x/\Psi(x))$ is the elasticity of function $\Psi(x)$ evaluated at the steady-state equilibrium. In addition, κ_n and ϵ_n are coefficients in the log-linearized net-worth equation of financial intermediaries: $\kappa_n = (1 - \gamma)(1 - \Gamma(\bar{\omega}))$ and $\epsilon_n = 1 + \epsilon_\psi - (\epsilon_\Gamma/\epsilon_\nu)\Gamma(\bar{\omega})/(1 - \Gamma(\bar{\omega}))$, $\epsilon_\Gamma = \Gamma'(\bar{\omega})\bar{\omega}/\Gamma(\bar{\omega})$, and $\epsilon_\nu = \nu'(\bar{\omega})\bar{\omega}/\nu(\bar{\omega})$. Here function $\nu(\bar{\omega})$ maps $\bar{\omega}$ into R_c/R , the ratio of the gross return of long-term bonds to the short-term return, so that $\nu(\bar{\omega}) = R_c/R$.

where ΩY_t is the income of the investment bank's manager and parameter γ is the death rate of investment bank's manager (that is introduced to prevent them from accumulating wealth enough not to rely on any banks for their investment funds).

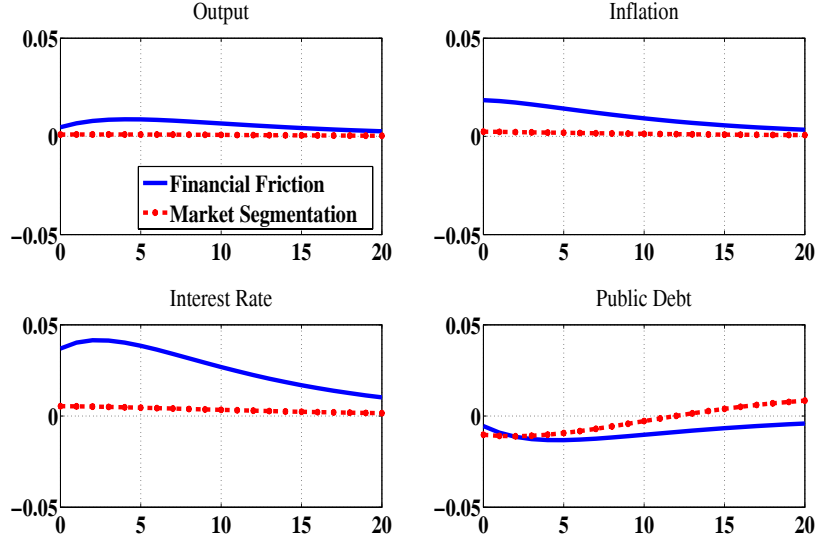
In order to derive the demand curve for deposits, the convenience yield of deposits is introduced into the utility function of households. Specifically, the preferences of households at period 0 are represented by the following equation:

$$\sum_{t=0}^{\infty} \beta^t E_0[u(c_t(h), 1 - n_t(h)) + v(\frac{D_t(h)}{P_t})] \quad (8.5)$$

where $D_t(h)$ is the household h 's nominal value of deposits and $v(D_t(h)/P_t)$ denotes the utility of having deposits. The budget constraint at period t of household h is then given by

$$c_t(h) + \frac{D_t(h)}{P_t R_t} + \frac{P_{c,t} B_{c,t}(h)}{P_t} \leq w_t n_t(h) - T_t + \frac{D_{t-1}(h)}{P_t} + \frac{(1 + \delta P_{c,t}) \sum_{k=1}^{\infty} \delta^{k-1} B_{c,t-k}(h)}{P_t} + \Phi_t \quad (8.6)$$

Figure 11: Dynamic Effects of Real Primary Surplus Shocks



Note: This figure shows impulse responses of aggregate variables when real budget surplus rises by one percent. The exogenous real primary surplus follows an AR(1) processes while its persistence parameter is set to be 0.5.

where T_t is the real transfer from households to managers of investment banks. In particular, the real transfer T_t is set to be $T_t = \Omega Y_t$. Although this assumption is arbitrarily made, it facilitates to have a well-defined evolution equation of the net-worth of investment managers. The household's utility optimization condition for deposits is also given by

$$\lambda_t = R_{d,t} \left(v' \left(\frac{D_t}{P_t} \right) + \beta E_t \left[\frac{\lambda_{t+1}}{\Pi_{t+1}} \right] \right) \quad (8.7)$$

where λ_t denotes the marginal utility of consumption at period t . This optimization condition serves as the demand curve for deposits given the market deposit rate. Since the commercial banking industry is perfectly competitive, the zero profit condition helps determine the market deposit rate. In other words, the market deposit rate is determined by solving the following zero profit condition for the commercial banking industry: $R_t(B_{s,t}/R_t + P_{c,t}K_{c,t} - N_{c,t}) = R_{d,t}D_t$.

The aggregate market-clearing condition for goods is

$$y_t = \frac{1}{1 - \Omega} (c_t + c_t^e + \mu G(\bar{\omega}_t) R_{c,t} P_{c,t-1} K_{c,t-1})$$

where C_t^e is the aggregate consumption of investment bank's managers and the third term is the aggregate amount of resources devoted to monitoring costs. The instantaneous government's flow budget constraint can be written as

$$\frac{B_{s,t}}{R_t} + P_{c,t} B_{c,t} = C_{p,t} + B_{s,t-1} - P_t s_t \quad (8.8)$$

where $C_{p,t}$ is the total nominal coupon payments at period t and $B_{c,t}$ is the nominal amount at period t of short-term securities. In addition, as explained in the previous section, the geometric decay of coupons over time leads to a simple evolution equation for $C_{p,t}$: $C_{p,t} = B_{c,t} + \delta C_{p,t-1}$. The aggregate nominal public debt at period t is defined as $B_t = B_{s,t}/R_t + P_{c,t}C_{p,t}$. The market clearing condition for long-term government securities requires $K_{c,t} = B_{c,t}$. The no-arbitrage condition leads to the following condition:

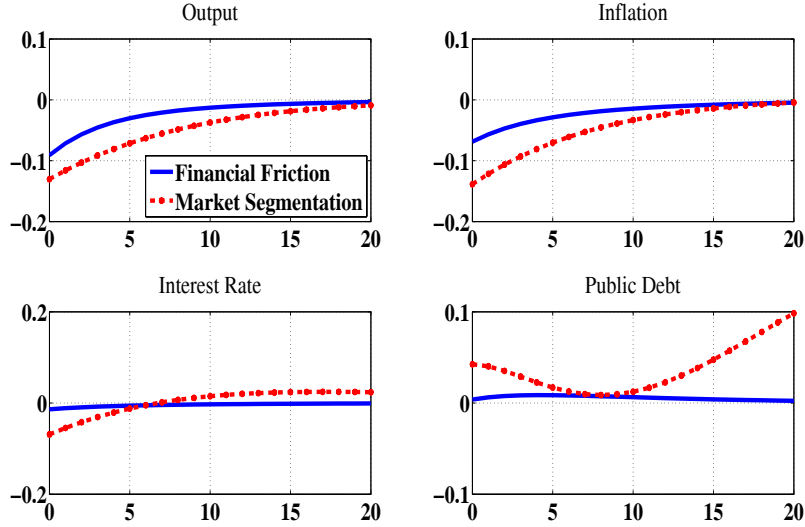
$$\beta E_t \left[\frac{\lambda_{t+1} R_{c,t+1}}{\lambda_t \Pi_{t+1}} \right] = 1 \quad (8.9)$$

where $R_{c,t+1}$ is defined as $R_{c,t+1} = (1 + \delta P_{c,t+1})/P_{c,t}$. The log-linearization of this equation around the steady state then leads to the IS curve equation.

Table 4 summarizes the log-linearized equilibrium conditions. The first equation is the IS curve, the second equation is the net-worth equation of financial intermediaries, and the third equation describes the demand curve of government's long-term bonds that is a log-linear approximation to equation (8.3). It should be mentioned that the log-linearization of the aggregate social resource constraint is $\hat{y}_t = s_c \hat{c}_t + s_e \hat{c}_t^e + \phi_t^m$ where s_c is the share of households' consumption, s_e is the consumption share of investment bank managers, and ϕ_t^m represents the log-linear approximation of the aggregate monitoring costs. In the calibration of this section, the share of resources devoted to monitoring costs and consumption of investment-bank managers is set to be insignificantly small (essentially close to zero). Hence, the consumption gap (that is defined as the logarithmic deviation of consumption in the sticky-price equilibrium from that of the flexible-price equilibrium) turns out to be equal to the output gap, which is reflected in the IS curve shown in Table 4.

Even with this approximation, the IS curve in this model seems to be quite different from the estimated IS curve. However, it will be demonstrated that this model generates impulse responses of aggregate variables similar to those shown in the reduced-form model of financial friction. Figure 11 shows the impulse responses of aggregate variables with respect to an exogenous rise in the real primary surplus. The straight line corresponds to the reduced form model of financial frictions, while the dotted line corresponds to the impulse responses in the market-segmentation model. As shown in Figure 11, an increase in the real primary surplus reduces the aggregate inflation and output. The main reason for this result is that when agents anticipate persistent increases in the budget surplus, they increase their demands for long-term government bonds and the external premium of long-term government securities rises. Hence, the real amount of short-term government bonds declines to meet persistent falls in the budget surplus. The increases in the external premium for holding long-term government bonds reduce the aggregate demand and therefore inflation drops as well. Figure 12 shows dynamic impacts of monetary policy shocks. The straight line corresponds to the reduced-form model of financial frictions, while the dotted line corresponds to the impulse

Figure 12: Dynamic Effects of Monetary Policy Shocks



Note: This figure shows impulse responses of aggregate variables with respect to an exogenous increase in the policy rate. The exogenous shock follows an AR(1) process while its persistence parameter is set to be 0.5.

responses in the market-segmentation model of government bonds. As shown in this figure, both inflation rate and output fall in this market-segmentation model. In sum, this simple model of market-segmentation generates equilibrium dynamics similar to those of the reduced-form model of financial friction (the case of $\sigma_b < 0$).

9 Conclusion

Based on the analysis of this paper, there are three points to be summarized in this conclusion. First, multiple distinct channels might exist in the way through which the public debt affects the aggregate demand. The financial friction channel of the public debt discussed in this paper may help explain how a rise in the public debt can repress the aggregate demand, especially in the era of persistently high levels of the public debt. Second, the main effective transmission mechanism of the public debt might not be fixed over time. Rather, it seems likely to change over time. Since the effective transmission mechanism of the public debt varies over time so that it is relatively difficult to capture, fiscal uncertainty might be enlarged during the period like the recent Great Recession. Third, once the financial friction channel of the public debt begins to work, it dramatically affects the nature of monetary and fiscal policy interaction. For example, to the extent which a significantly negative elasticity of the public debt in the IS curve helps break down the conventional wisdom about consequences of keeping the Taylor principle and pursuing non-Ricardian fiscal policy regime

at the same time for the existence of the unique equilibrium path.

Table 5: Parameter Values

parameter	value	definition
β	0.99	time discount factor
δ	0.79	maturity parameter
ϕ_π	(1.5,0.2)	coefficient of inflation
ϕ_x	(0.5,0.0)	coefficient of output
κ_x	0.035	slope of the Phillips curve
σ	1	elasticity of inter-temporal substitution
c	0.0072	convenience yield at the steady state
ϵ_s	1	elasticity of the marginal utility of government's short-term bonds
ω_s	0.5	share of short-term government debt
ϕ_b	0.6	elasticity of long-term debt with respect to the total debt
γ	0.0272	death rate of investment managers

Appendix

A Calibration

In this appendix, I focus on the description of how to set parameter value for long-term government bonds and the market-segmentation model. The evolution equation of $c_{p,t}$ implies a steady-state relation between c_p and b_c : $c_p = b_c/(1-\delta)$. In addition, the steady-state price of long-term bonds is $P_c = 1/(1-\beta\delta)$. Along with these two relations, the government's budget constraint at the steady state can be written as $(b_s/R)(R-1) + \delta(1-\beta)c_p P_c = s$. Hence, rearranging this equation leads to the following equation: $s/b = \omega_s(R-1) + \delta(1-\beta)(1-\omega_s)$. Given this result, the value of ξ in the log-linearized government's budget constraint is determined once values of ω_s , R , β , and δ are set. The values of parameters ω_s , R , β , and δ are then reported in Table 5.

In the market segmentation model, it would be worthwhile to discuss how to choose coefficients κ_n and ϵ_n of the log-linearized net-worth equation of financial intermediaries. Specifically, their definitions are $\kappa_n = (1-\gamma)(1-\Gamma(\bar{\omega}))$ and $\epsilon_n = 1 + \epsilon_\psi - (\epsilon_\Gamma/\epsilon_\nu)\Gamma(\bar{\omega})/(1-\Gamma(\bar{\omega}))$, $\epsilon_\Gamma = \Gamma'(\bar{\omega})\bar{\omega}/\Gamma(\bar{\omega})$, and $\epsilon_\nu = \nu'(\bar{\omega})\bar{\omega}/\nu(\bar{\omega})$. In order to determine values of these elasticities, it should be noted that the optimization conditions of the debt contract problem specified in equations (8.1) and (8.2) can be used to derive $\nu(\bar{\omega}) = R_c/R$ where function $\nu(\bar{\omega})$ is defined as follows:

$$\nu(\bar{\omega}) = \frac{\lambda(\bar{\omega})}{1 - \Gamma(\bar{\omega}) + \lambda(\bar{\omega})(\Gamma(\bar{\omega}) - \mu G(\bar{\omega}))} \quad \lambda(\bar{\omega}) = \frac{\Gamma'(\bar{\omega})}{\Gamma(\bar{\omega}) - \mu G'(\bar{\omega})}.$$

In Bernanke, Gertler, and Gilchrist (1999, hereafter BGG), parameters were chosen to match three steady-state conditions: First, a risk spread between risky and risk-less rates = two hundred basis

points, approximately the historical average spread between the prime lending rate and the six-month Treasury bill rate; Second, an annualized business failure rate = three percent; Third, a ratio of capital to net worth = 2 (or equivalently, a leverage ratio of 0.5). In order to produce similar parameter values for the market segmentation model, the death rate of managers is set to be $\gamma = 0.0272$ (quarterly) and the monitoring cost parameter is $\mu = 0.12$.

References

- Acharya, Viral V., Itamar Drechsler, and Philipp Schnabl. 2010. "A Pyrrhic Victory? - Bank Bailouts and Sovereign Credit Risk." New York University. Unpublished Manuscript. August.
- Bernanke, Ben, Mark Gertler, and Simon Gichrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In: Taylor J., Woodford M. (Eds.), *Handbook of Macroeconomics*. Elsevier Science B.V. pp. 1343-1393.
- Bi, Huixin. 2010. "Sovereign Default Risk Premia, Fiscal Limits, and Fiscal Policy." Unpublished Manuscript. Bank of Canada. October.
- Blanchard, Olivier and Alessandro Missale. 1994. "The Debt Burden and Debt Maturity." *American Economic Review*. 84 (1). March. pp. 309-319.
- Campbell, John Y., and Robert J. Shiller. 1987. "Cointegration and Tests of Present Value Models". *Journal of Political Economy* 95. pp. 1062-1088.
- Canzoneri, Matthew, Robert Cumby, and Behzad Diba. 2011. "The Interaction between Monetary and Fiscal Policy." In: Friedman B., Woodford M. (Eds.), *Handbook of Monetary Economics* 3B. North-Holland. Amsterdam.
- Greenwood, Robin, and Vayanos, Dimitri. 2010. "Bond Supply and Excess Bond Returns." Unpublished Manuscript. Harvard University and London School of Economics. August.
- Davig, Troy, Eric Leeper, and Todd Walker. 2011. "Inflation and the Fiscal Limit." *European Economic Review* 55. pp. 31-47.
- Gali, Jordi and Mark Gertler. 1999. "Inflation Dynamics: A Structural Econometric Approach," *Journal of Monetary Economics*, 44 (2), October, pp. 195-222.
- Garcia-Cicco, Javier, Roberto Pancrazi, and Martin Uribe. 2011. "Real Business Cycles in Emerging Countries." *American Economic Review* 100 (5). pp. 2510-2531.
- Krishnamurthy, Arvind, and Vissing-Jorgensson. 2010. "The Aggregate Demand for Treasury Debt." Northwestern University. Unpublished Manuscript.
- Fernandez-Villaverde, Jesus. 2010. "Fiscal Policy in a Model with Financial Frictions." *American Economic Review* 100 (May). pp. 35-40.
- Hall, George and Thomas Sargent. 2010. "Interest Rate Risk and Other Determinants of Post World War II U.S. Government Debt/GDP Dynamics." *American Economic Journal: Macroeconomics*.
- Jung T., Y. Teranish, and T. Watanabe. 2005. "optimal Monetary Policy at the Zero-Interest-Rate Bound." *Journal of Money, Credit, and Banking* 37 (5): pp. 813 - 835.
- Lane, Philip R. 2011. "Fiscal Policy and Financial Stability." Trinity College Dublin. Unpublished Manuscript. May.
- Laubach, Thomas. 2009. "New Evidence on the Interest Rate Effects of Budget Deficits and Debt." *Journal of the European Economic Association* 7 (4). pp. 858-885.
- Leeper, Eric M. 1991. "Equilibria under Active and Passive Monetary Policies." *Journal of*

Monetary Economics 27. pp. 129-147.

Leeper, Eric M. 2010. "Monetary Science, Fiscal Alchemy." Unpublished Manuscript. Indiana University. October.

Leeper, Eric M. and Todd Walker, 2011. "Perceptions and Misperceptions of Fiscal Inflation." Unpublished Manuscript. Indiana University. June.

Levin, A. T., D. Lopez-Salido, E. Nelson, and T. Yun. 2010. "Limitations on the Effectiveness of Forward Guidance at the Zero Lower Bound." International Journal of Central Banking, Vol. (6) No. 1. pp. 143 - 189.

Levin, Andrew T. and Tack Yun. 2011. "Durable Goods and Persistent Recession." Unpublished Manuscript. Seoul National University. May.

Mishkin, Frederic S. The Economics of Money, Banking, and Financial Markets. Ninth Edition. Pearson. Boston.

Reinhart, Carmen and Kenneth Rogoff. 2011. "A Decade of Debt." NBER Working Paper No. 16827.

Sbordone, Argia. 2002. "Prices and Unit Labor Costs: A New Test of Price Stickiness", Journal of Monetary Economics, vol. 49 (2). pp. 235-456.

Shabert, Andreas. 2010. "Monetary Policy under a Fiscal Theory of Sovereign Default." Journal of Economic Theory 145. pp. 860-868.

Sims, Christopher A. 2011. "Modeling the Influence of Fiscal Policy on Inflation." Presentation Slides at the Conference held at Seoul National University in Korea, May.

Uribe, Martin. 2006. "A Fiscal Theory of Sovereign Risk." Journal of Monetary Economics 53. pp. 1857-1875.

Woodford, Michael. 1995. "Price Level Determinacy without Control of a Monetary Aggregate." Carnegie-Rochester Conference Series on Public Policy 43. pp. 1-46.

Woodford, Michael. 1998. "Control of the Public Debt: A Requirement for Price Stability?" In: Calvo G., King M. (Eds.), The Debt Burden and Monetary Policy. Macmillan, London.

Woodford, Michael. 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University. Princeton NJ.