Optimal Nonlinear Taxation: Two Wage-Types

Wage rates $w_2 > w_1$

- Government observes income $y(= w\ell)$, not ℓ or w
- n_1, n_2 and $u(c, \ell)$ are common knowledge
- Common consumer budget constraint is $c = y - T(y)$
- Government observes y and $T(y)$, so knows c

Transform household utility: $v^i(c, y) \equiv u(c, y/w_i)$
where $v^i_c = u_c, \quad v^i_y = u_\ell / w_i$

Given (c, y) with $y > 0$, $\ell_1 > \ell_2$, so $y^2(c, y) > v^1(c, y)$

$$\left.\frac{dc}{dy}\right|_{v^i} = -\frac{v^i_y}{v^i_c} = -\frac{u_\ell}{u_c}\frac{1}{w_i} \implies \left.\frac{dc}{dy}\right|_{v^1} > \left.\frac{dc}{dy}\right|_{v^2}$$

(Single-crossing property) FIGURE 1

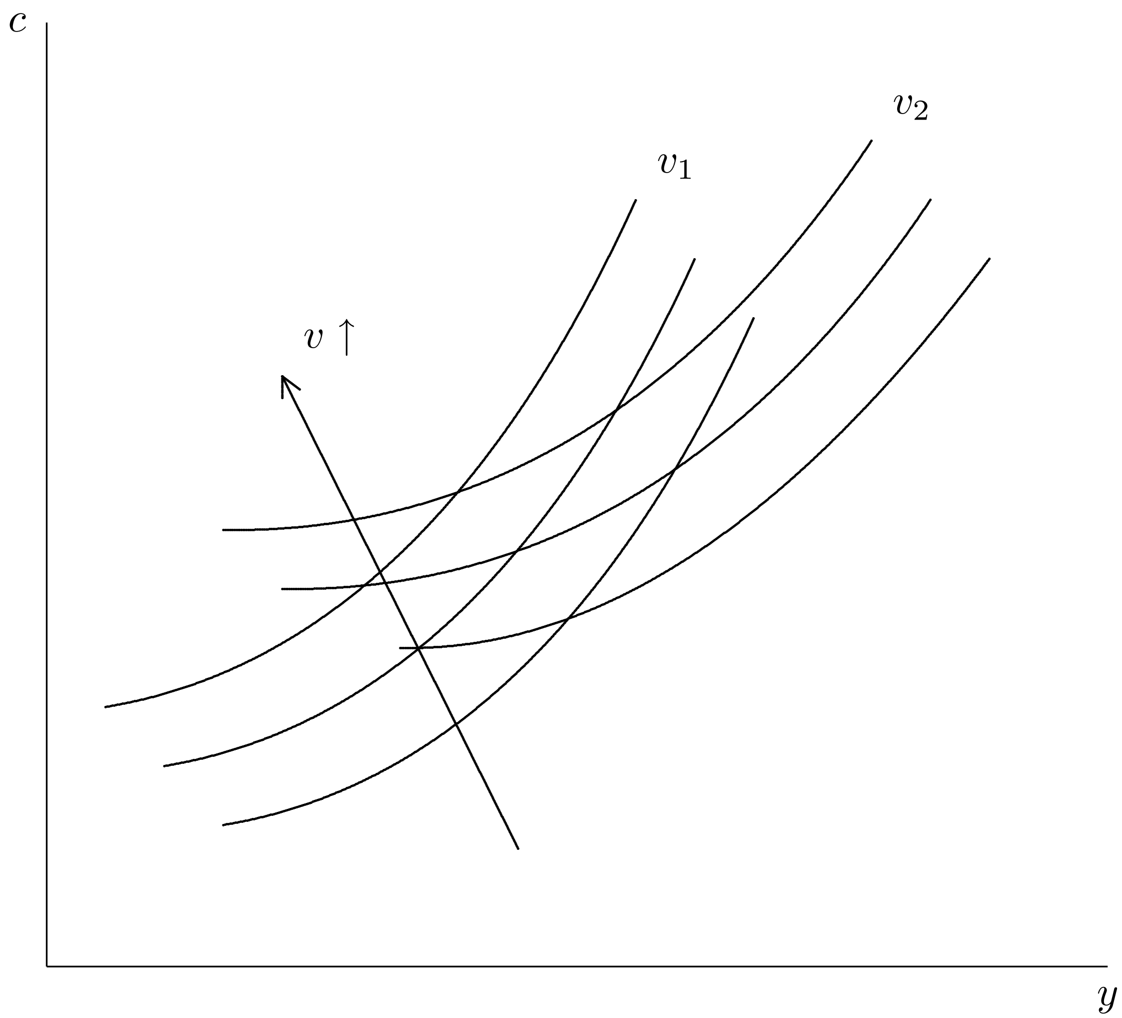


Figure 1

Incentive Compatibility (IC)

In redistributing from type-2's to type-1's, IC requires

$$v^2(c_2, y_2) \geq v^2(c_1, y_1) > v^1(c_1, y_1) \quad \text{for} \quad y > 0$$

This implies:

- Type-2's must at least as well off as type-1's in optimum
- IC becomes binding on First-Best UPF where $v^2 > v^1$
- Non-distorting taxes are possible where IC does not bind: FIGURE 2A
- If IC strictly binds, taxes must be distorting: FIGURE 2B
- FIGURE 2C, IC just binds

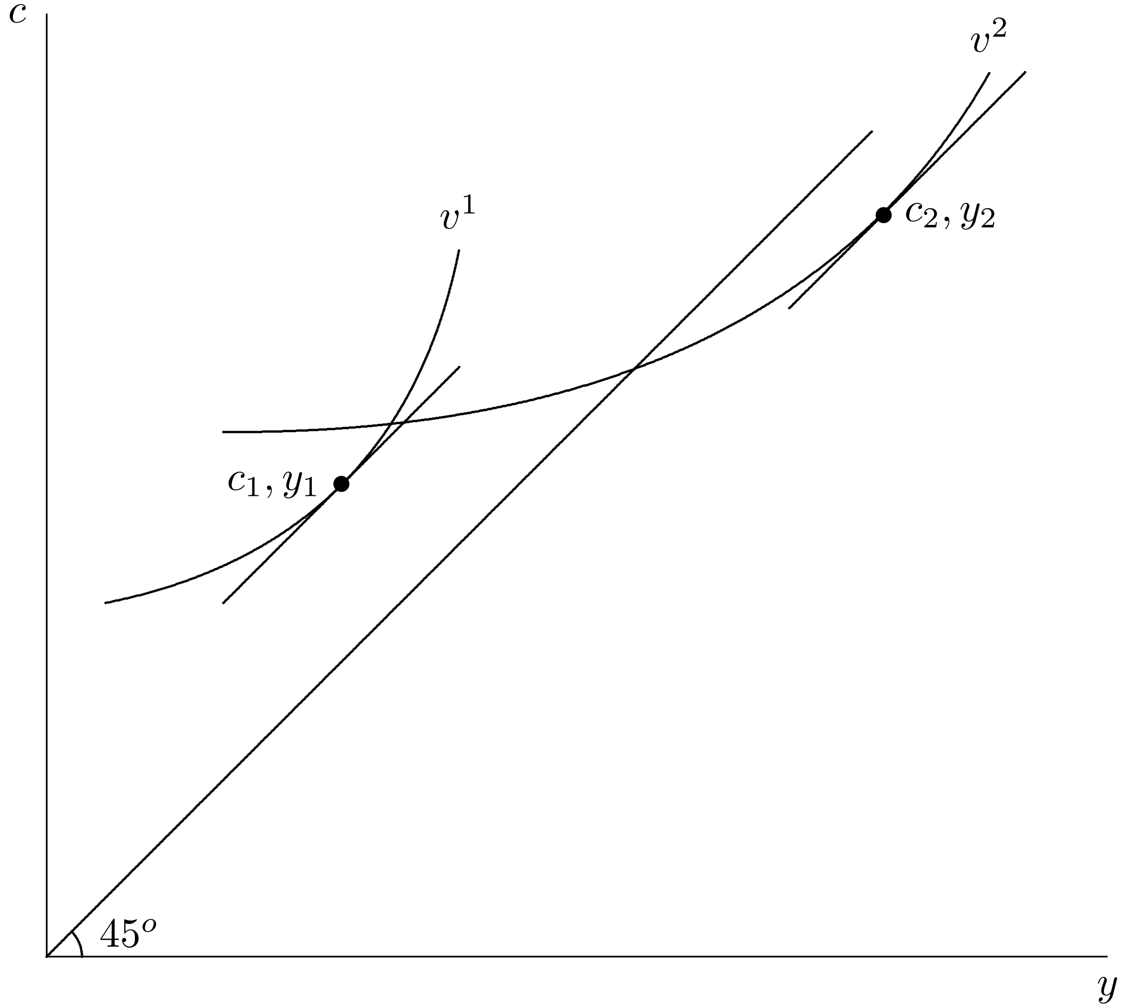


Figure 2A

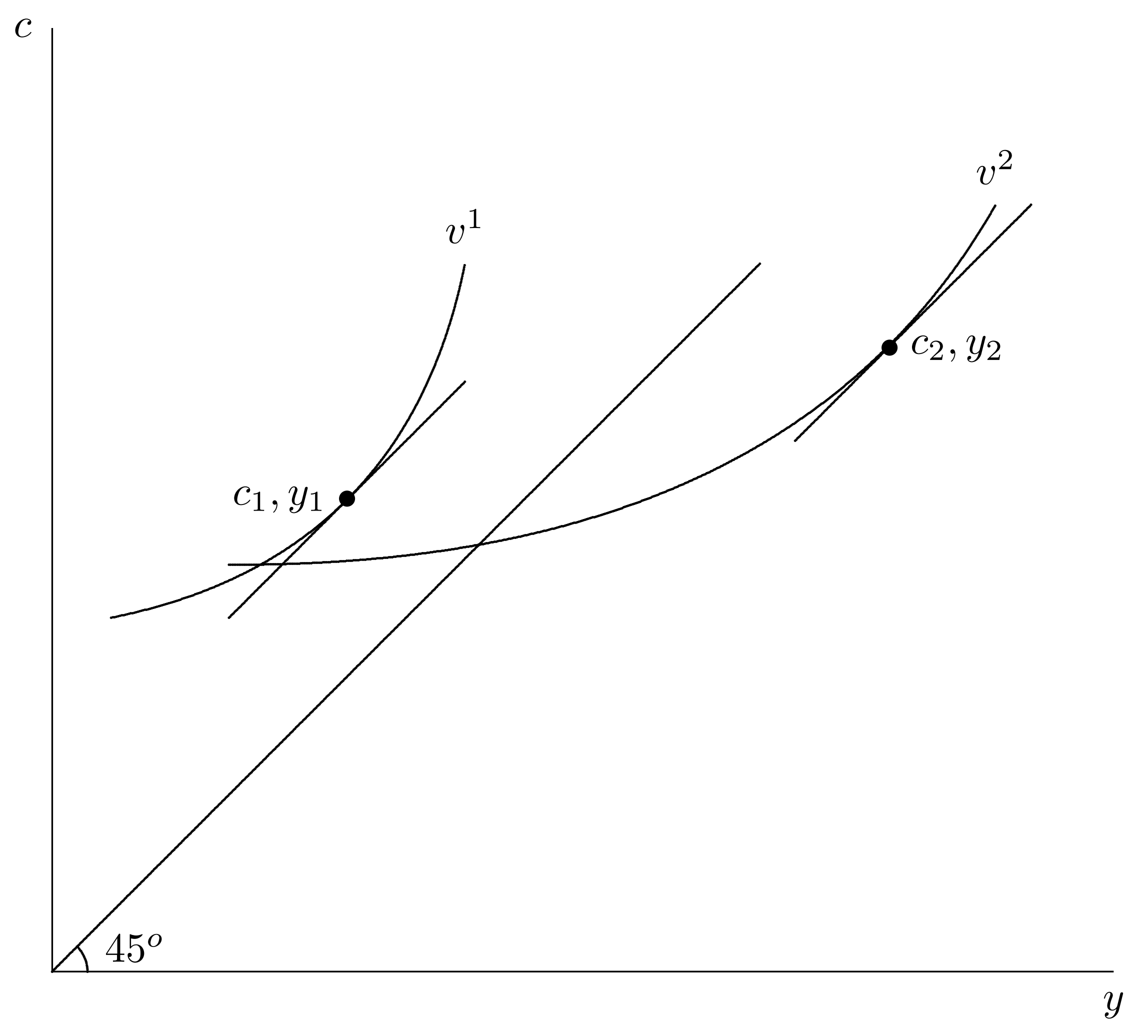


Figure 2B

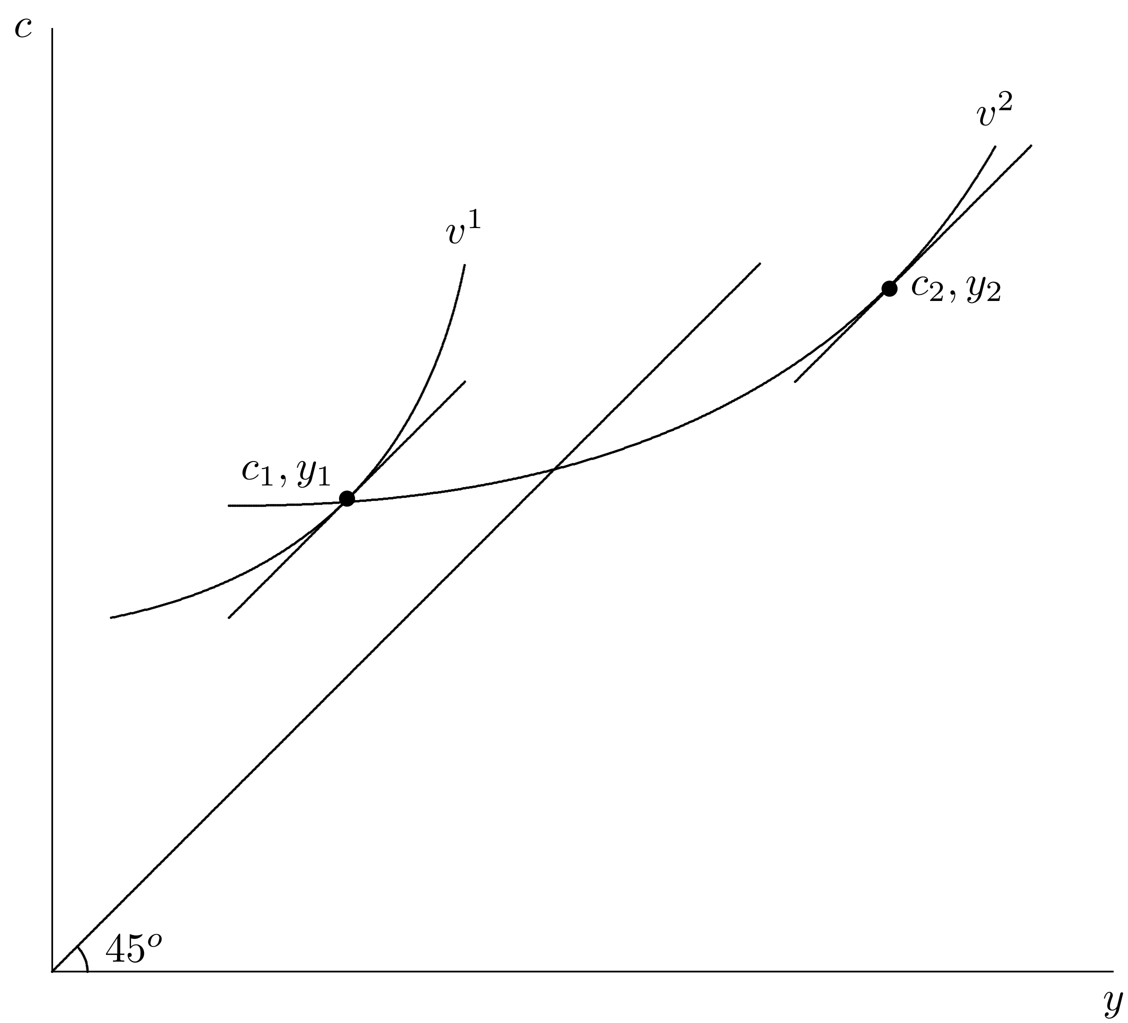


Figure 2C

Government's Optimal Income Tax Problem

Government selects a tax system $T(y)$ to maximize an objective function subject to a budget constraint

Equivalent direct approach: Select bundles (c_1, y_1, c_2, y_2) to maximize an objective function subject to:

- Budget constraint:

  $n_1 T(y_1) + n_2 T(y_2) = n_1(y_1 - c_1) + n_2(y_2 - c_2) = R$

- IC constraints:

  $v^2(c_2, y_2) \geq v^2(c_1, y_1)$

  $v^1(c_1, y_1) \geq v^1(c_2, y_2)$

Households choose preferred bundle

Implement the direct solution via $T(y)$

Which Incentive Constraint?

Both constraints cannot be binding (Single-Crossing Property):

- When $v^2(c_2, y_2) = v^2(c_1, y_1)$, $v^1(c_1, y_1) > v^1(c_2, y_2)$
- When $v^1(c_1, y_1) = v^1(c_2, y_2)$, $v^2(c_2, y_2) > v^2(c_1, y_1)$

If government has non-negative aversion to inequality, IC on Type 2's will bind in an optimum:

$$v^2(c_2, y_2) = v^2(c_1, y_1), \quad v^1(c_1, y_1) > v^1(c_2, y_2)$$

(since First-Best outcome is between Maximin and Utilitarian where $v^1 \geq v^2$)

Notation: $\widehat{v}^2(c_1, y_1)$ denotes utility of type-2 when mimicking type-1's (c, y) bundle

Government Pareto-Optimizing Problem

Pareto-optimizing Lagrangian expression:

$$\mathcal{L} = v^1(c_1, y_1) + \rho\left[v^2(c_2, y_2) - \overline{v}^2\right] + \gamma\left[v^2(c_2, y_2) - \widehat{v}^2(c_1, y_1)\right]$$

$$+\lambda\left[n_1(y_1 - c_1) + n_2(y_2 - c_2) - R\right] \tag{1}$$

First-order conditions:

$$v_c^1 - \gamma \widehat{v}_c^2 - \lambda n_1 = 0 \tag{2}$$

$$v_y^1 - \gamma \widehat{v}_y^2 + \lambda n_1 = 0 \tag{3}$$

$$\rho v_c^2 + \gamma v_c^2 - \lambda n_2 = 0 \tag{4}$$

$$\rho v_y^2 + \gamma v_y^2 + \lambda n_2 = 0 \tag{5}$$

$\Longrightarrow$ Point on Second-Best UPF

Implicit Marginal Tax Rate

Given $T(y)$, type i maximizes $v^i(c, y)$ subject to budget constraint $c = y - T(y)$, or:

$$\max_y \quad v^i\big(\underbrace{y - T(y)}_{c}, y\big)$$

First-order condition is:

$$\big(1 - T'(y_i)\big)\, v_c^i + v_y^i = 0$$

or:

$$T'(y_i) = 1 + \frac{v_y^i}{v_c^i} \leq 1$$

This is the marginal tax rate on type i

Properties of Second-Best Optimum

1. *If IC not binding ($\gamma = 0$), $T'(y_1) = T'(y_2) = 0$*

FOCs yield: $$MRS^1_{cy} = -\frac{v^1_y}{v^1_c} = 1 = MRS^2_{cy} = -\frac{v^2_y}{v^2_c}$$

2. *Equilibrium must be a separating: $(c_1, y_1) \neq (c_2, y_2)$*
From a pooling allocation, can move one bundle along 45^o and make one person better off without violating constraints

—FIGURE 3—

3. *Marginal tax rate on high-ability persons is zero: $T'(y_2) = 0$*
Divide (5) by (4):

$$-\frac{v^2_y}{v^2_c} = 1$$

—FIGURE 4—

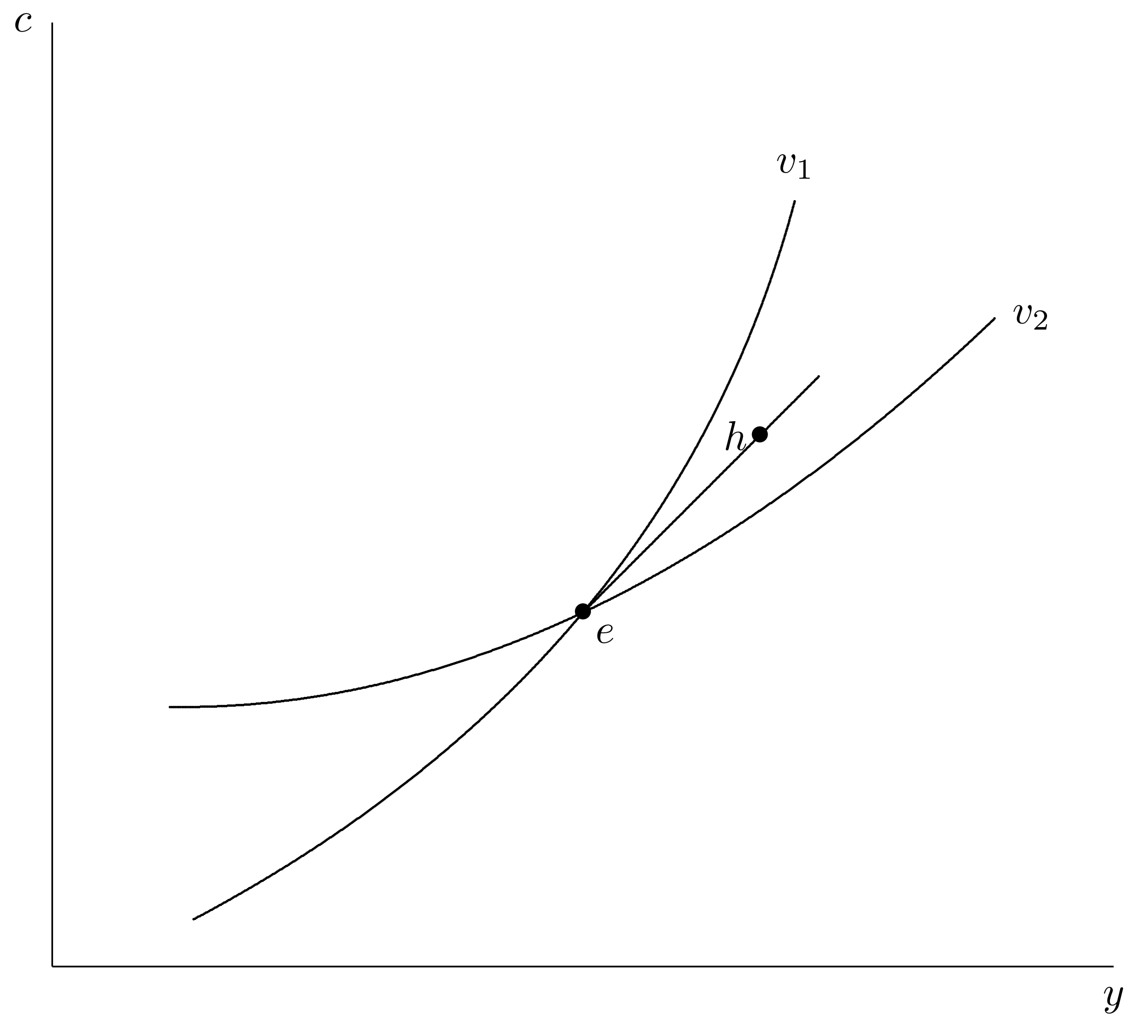


Figure 3

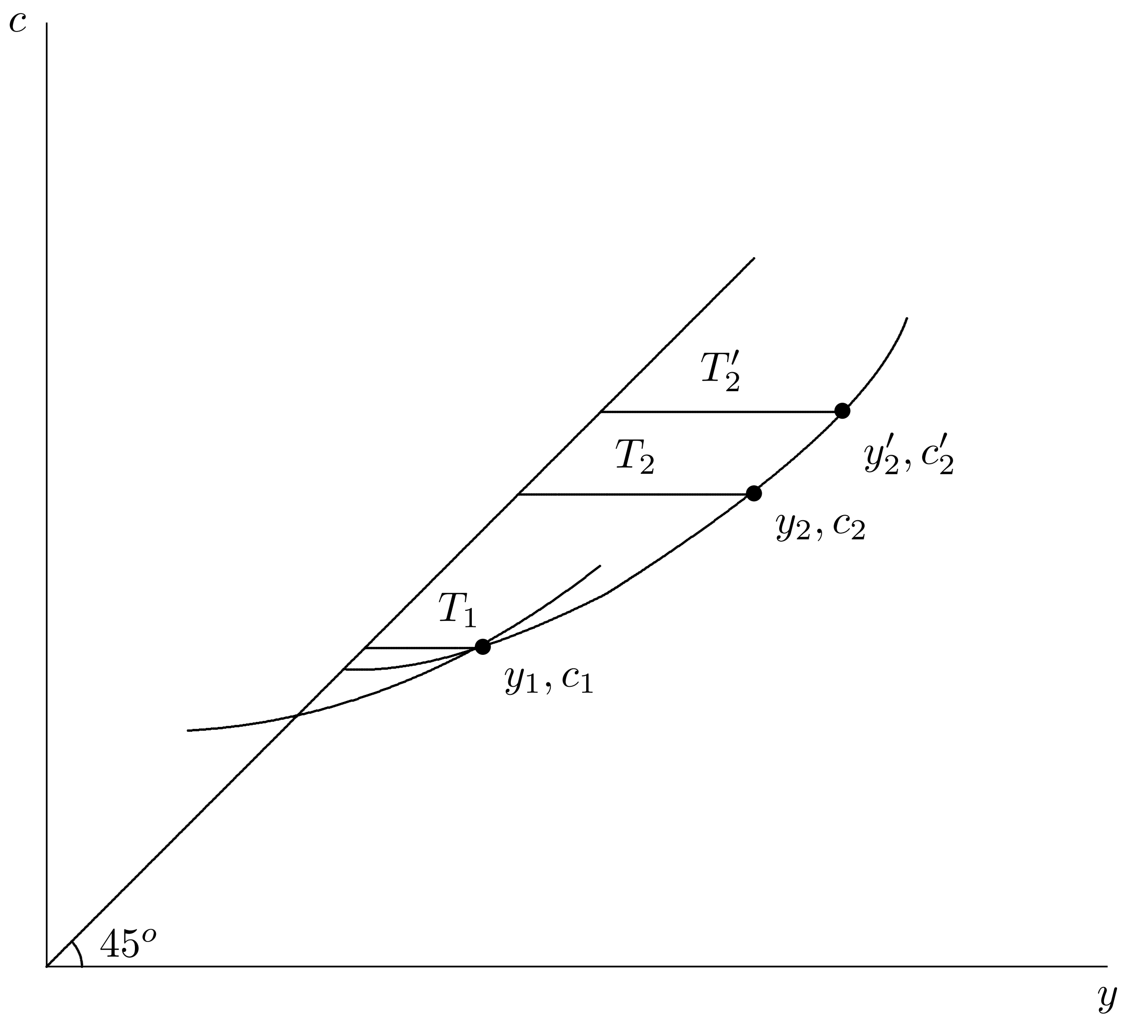


Figure 4

Properties, continued

4. *If IC binding, $T'(y_1) > 0$*
Divide (3) by (2):

$$-\frac{v_y^1}{v_c^1} = \frac{-\gamma\widehat{v}_y^2 + \lambda n_1}{\gamma\widehat{v}_c^2 + \lambda n_1} = \frac{-\frac{\widehat{v}_y^2}{\widehat{v}_c^2}k + 1}{k+1}$$

where $k = \gamma\widehat{v}_c^2/(\lambda n_1) > 0$. Since $0 < -\widehat{v}_y^2/\widehat{v}_c^2 < 1$ at (c_1, y_1),

$$0 < -\frac{v_y^1}{v_c^1} < 1 \quad \Longrightarrow \quad 0 < T'(y_1) < 1$$

5. *As $\overline{v}^2$ is reduced, v^1 increases until Maximin solution*
   - Maximin may be interior where $v^2 > v^1$ (FIGURE 5A)
   - Maximin may be corner where $\ell_1 = 0, v^2 = v^1$ (FIGURE 5B)

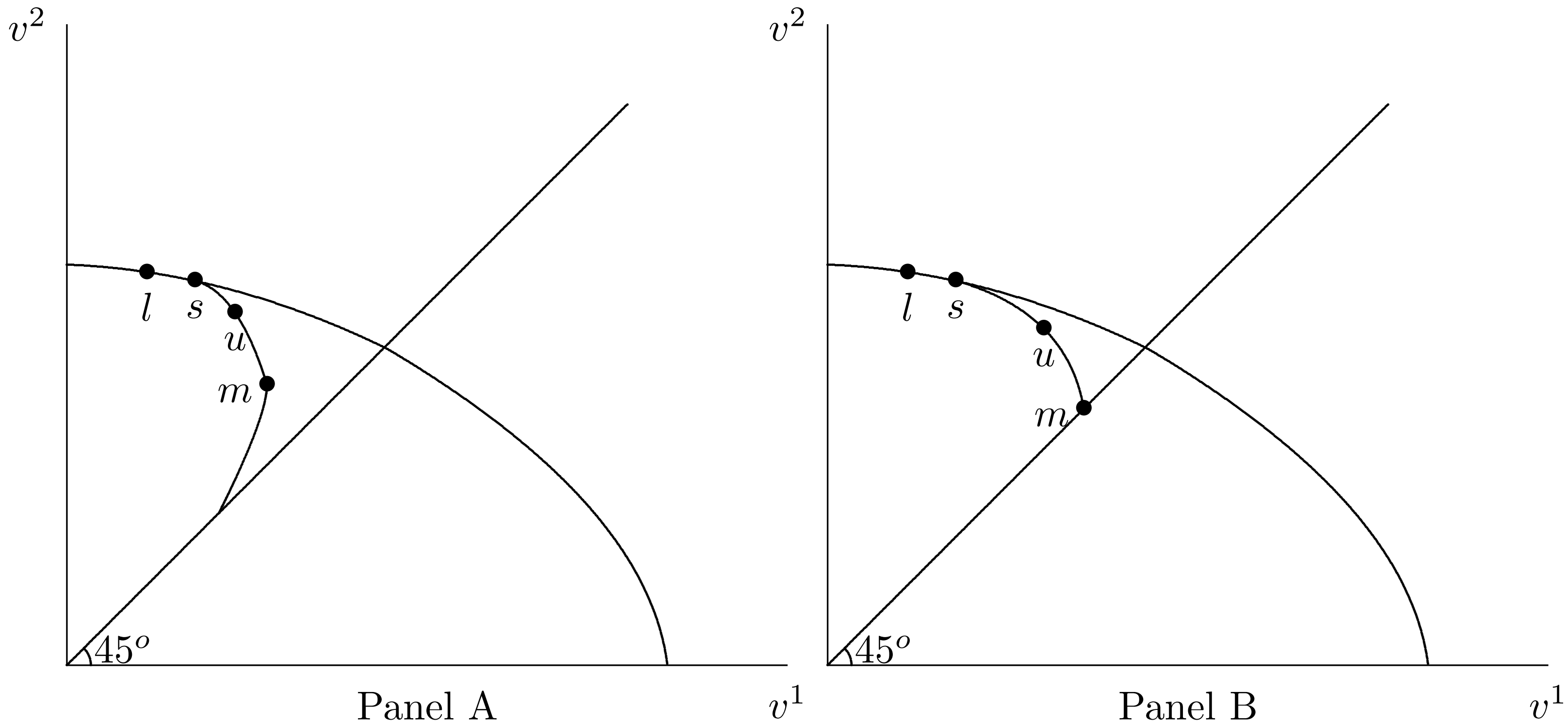


Figure 5

Properties, continued

6. *Optimal implemented by many tax structures:* FIGURE 6
In the optimum:

$$c_2 > c_1, \quad y_2 > y_1, \quad v^2 > v^1, \quad T(y_2) > T(y_1)$$

but
$$\frac{T(y_2)}{y_2} \gtreqless \frac{T(y_1)}{y_1}$$

$\Longrightarrow$ Tax system can be progressive or regressive

7. *Linear progressive taxation not efficient*
Incentive constraint not binding

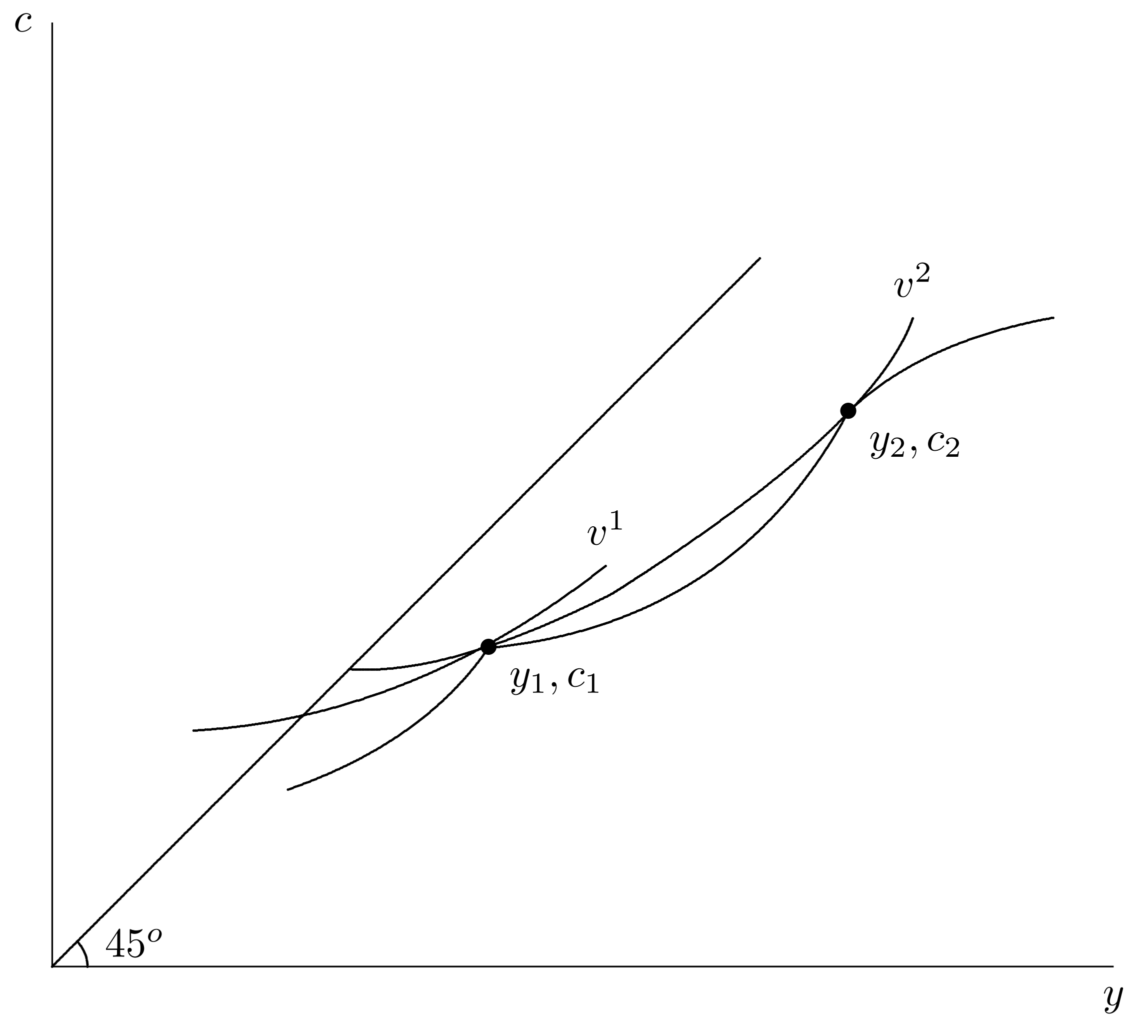
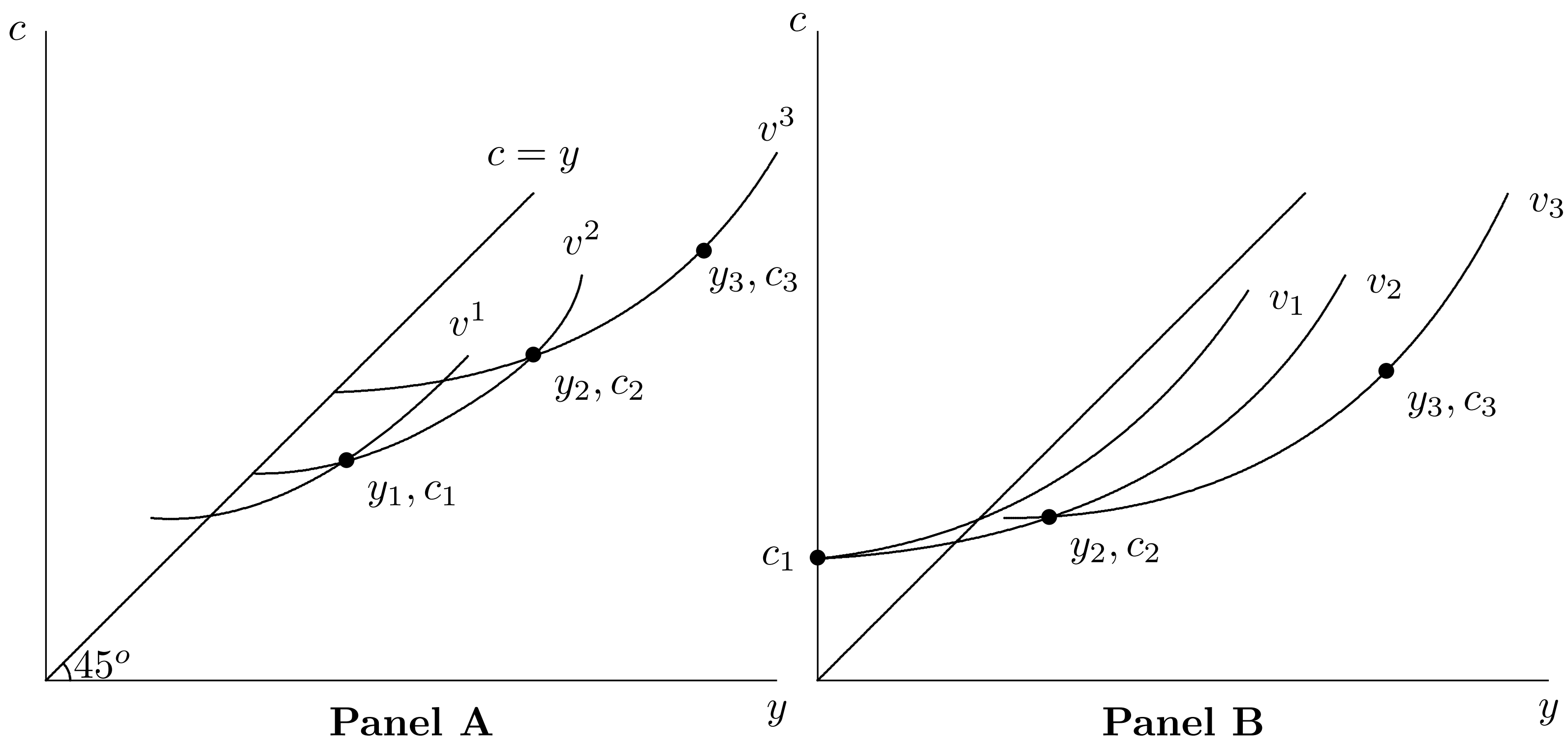


Figure 6

More than Two Ability Types: $w_3 > w_2 > w_1$

Characteristics of optimal tax solution (Guesnerie-Seade):

- IC constraint binding on next lowest type only (FIGURE 7A)
- Lowest ability type(s) may not work (FIGURES 7B, 7C)
- Equilibrium may be partial pooling (FIGURE 8)
- Marginal tax rate zero at the top ($T'(y_3) = 0$)
- Marginal tax rates for $i = 1, 2$ between zero and one
- Optimal allocation satisfies:
  $c_i > c_{i-1},\ y_i > y_{i-1},\ v^i > v^{i-1},\ T(y_i) > T(y_{i-1})$
- Tax can be progressive or regressive



c
c = y
v^3
v^2
v^1
y_3, c_3
y_2, c_2
y_1, c_1
45^o
y
Panel A
c
v_3
v_1
v_2
y_3, c_3
c_1
y_2, c_2
y
Panel B

Figure 7

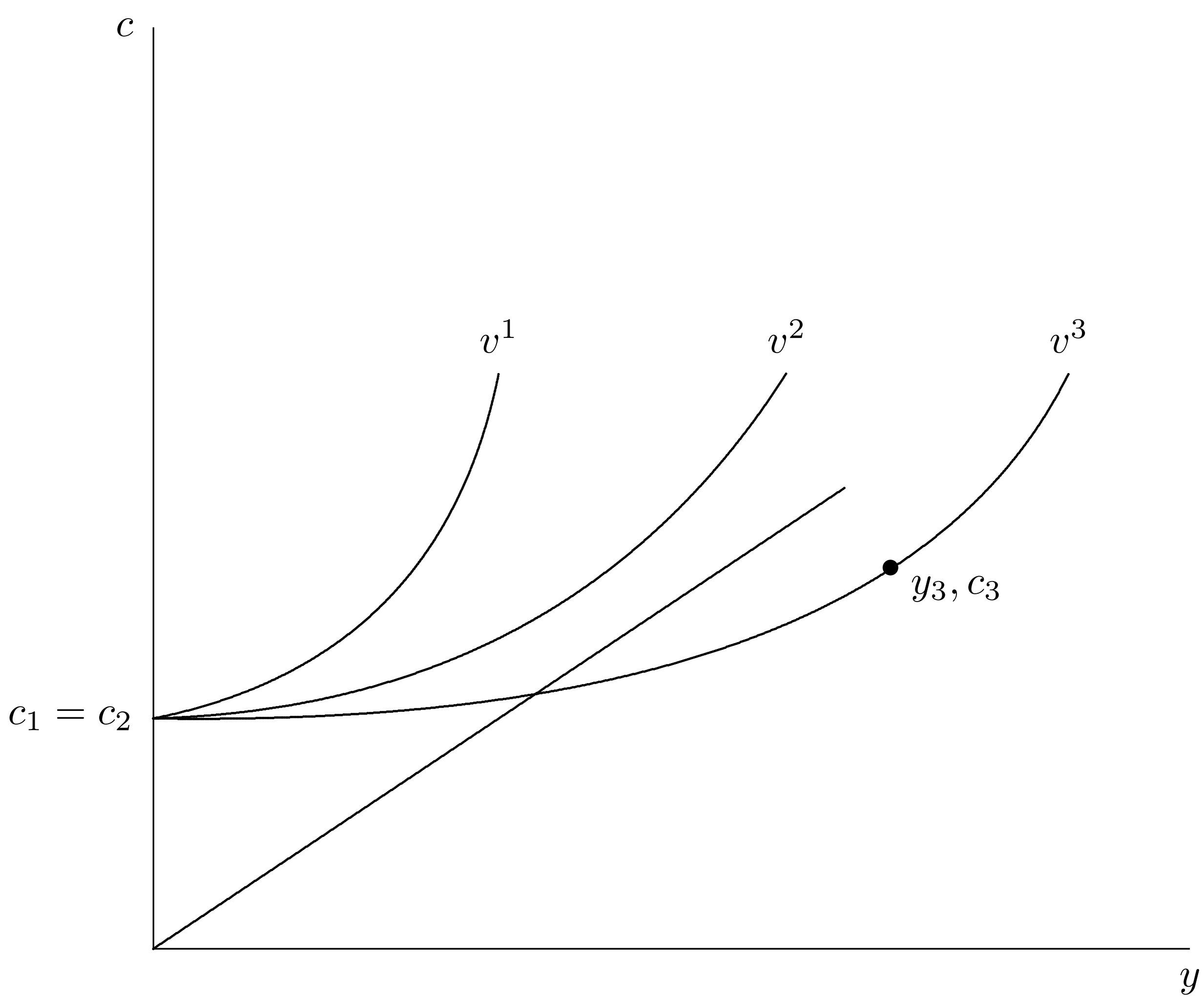


Figure 7, Panel C

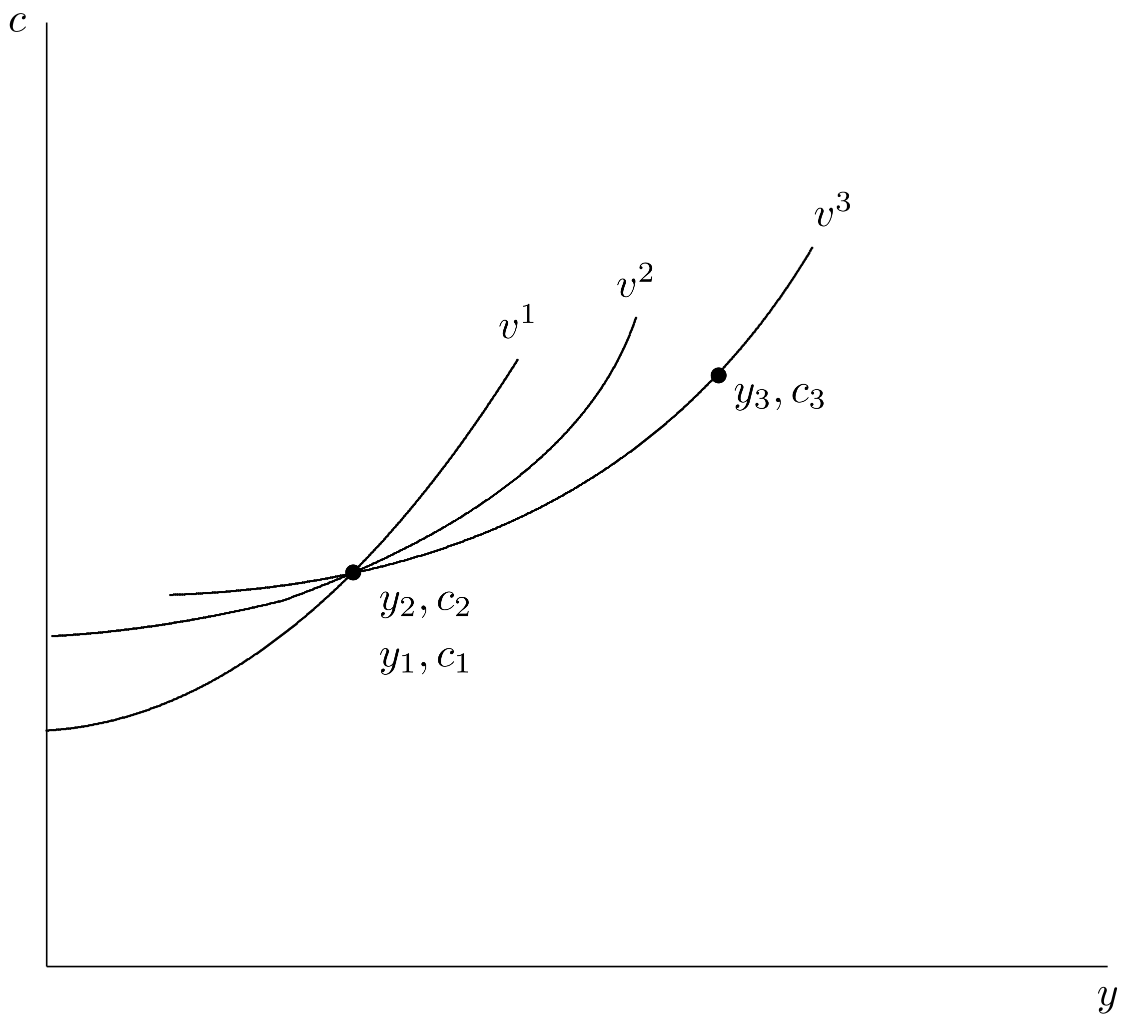


Figure 8

Continuous Wage Distribution (Mirrlees)

Distribution of abilities:

$$F(w), f(w) = F'(w), \quad w \in [\underline{w}, \overline{w}], \ \underline{w} \geq 0, \ \overline{w} \leq \infty$$

Utility: $u(w) = v\big(c(w), y(w), w\big)$

Incentive compatibility:

$$u(w) = v\big(c(w), y(w), w\big) \geq v\big(c(w'), y(w'), w\big), \quad \forall w'$$

$$\Longrightarrow u(w) = \max_{w'} v\big(c(w'), y(w'), w\big)$$

So, applying Envelope Theorem:

$$\Longrightarrow \ \dot{u}(w) = v_w\big(c(w), y(w), w\big)$$

This is the first-order incentive constraint (FOIC)

(An SOIC must also be satisfied: $\dot{y}(w) \geq 0$; we assume it is in what follows)

The Optimal Income Tax Problem

Max $\int_{\underline{w}}^{\overline{w}} W\big(u(w)\big) f(w) dw$ (SWF) subject to

$\int_{\underline{w}}^{\overline{w}} \big(y(w) - c(w)\big) f(w) dw \geq R$ (Budget constraint)

$\dot{u}(w) = v_w\big(c(w), y(w), w\big)$ (FOIC)

But, $u(w), y(w), c(w)$ satisfy $u(w) = v(c(w), y(w), w) \Rightarrow$
Solve for $c(w) = c\big(y(w), u(w)\big)$, so government problem becomes:

Max $\int_{\underline{w}}^{\overline{w}} W\big(u(w)\big) f(w) dw$ (SWF) subject to

$\int_{\underline{w}}^{\overline{w}} \Big(y(w) - c(y(w), u(w))\Big) f(w) dw \geq R$ (Budget constraint)

$\dot{u}(w) = v_w\big(c(y(w), u(w)), y(w), w\big)$ (FOIC)

$y(w)$ is control variable, $u(w)$ is state variable

Additively Separable Case

Utility $u(w) = v\big(c(w), y(w), w\big) = u\big(c(w)\big) - h\big(y(w)/w\big)$, so

$$\frac{\partial c(w)}{\partial u(w)} = \frac{1}{v_c(\cdot)} = \frac{1}{u'(c(w))}; \quad \frac{\partial c(w)}{\partial y(w)} = -\frac{v_y(\cdot)}{v_c(\cdot)} = \frac{h'(\ell(w))}{wu'(c(w))}$$

Incentive constraint:
$\dot{u}(w) = v_w\big(c(w), y(w), w\big) = h'(\ell(w))\ell(w)/w$

Hamiltonian: $\mathcal{H} = W(u(w))f(w) + \lambda\big(y(w)$
$-c(y(w), u(w))\big)f(w) + \pi(w)h'(\ell(w))\ell(w)/w$

Necessary conditions (deleting w's):

$$\frac{\partial \mathcal{H}}{\partial y} = \lambda\Big(1 - \frac{\partial c}{\partial y}\Big)f + \pi\Big(\frac{h'}{w^2} + \frac{\ell h''}{w^2}\Big) = 0 \qquad (1)$$

$$\frac{\partial \mathcal{H}}{\partial u} = W'f - \lambda\frac{\partial c}{\partial u}f = -\dot{\pi} \qquad (2)$$

Transversality conditions: $\pi(\underline{w}) = \pi(\overline{w}) = 0$

Interpretation

Integrate (2) using $\pi(\overline{w}) = 0$:

$$\pi(w) = \int_{w}^{\overline{w}} \Big(\frac{W'}{\lambda} - \frac{1}{u'}\Big)\lambda dF \;\; < 0 \text{ for } \underline{w} < w < \overline{w} \qquad (3)$$

From (1), using $T' = 1 - \partial c/\partial y$:

$$T' = -\frac{\pi}{\lambda f}\Big(\frac{h'}{w^2} + \frac{\ell h''}{w^2}\Big) \;\; > 0 \text{ for } \underline{w} < w < \overline{w}$$

and $T'(y(\underline{w})) = T'(y(\overline{w})) = 0$, assuming no bunching

(With bunching at bottom, $T' > 0$ at end of bunching range)

From household problem, $(1 - T')u' = h'/w$. From (1) and (3):

$$\frac{T'}{1 - T'} = u' \int_{w}^{\overline{w}} \Big(\frac{1}{u'} - \frac{W'}{\lambda}\Big) dF \cdot \frac{1 + \ell h''/h'}{wf} \qquad (4)$$

Quasilinear case

Let $u(c, \ell) = c - h(\ell) = c - \ell^{1+1/\epsilon}/(1+1/\epsilon)$,
where $c = w\ell - T(w\ell)$

From consumer problem, we obtain $\ell = \big((1-T')w\big)^{\epsilon}$

Equivalently, $h''\ell/h' = 1/\epsilon$

Using $u' = 1$, (4) may be written:

$$\frac{T'}{1-T'} = \frac{1+\epsilon}{\epsilon} \cdot \frac{\int_w^{\overline{w}} (1 - W'(\tilde{w})/\lambda) dF(\tilde{w})}{1-F(w)} \cdot \frac{1-F(w)}{wf(w)}$$

$$= A(w) \cdot B(w) \cdot C(w) \quad \text{(Diamond 1998)}$$

The first term is an efficiency term; the second is an equity term; the third captures the proportion of the population above w

$T'/(1-T')$ is the marginal income tax rate in terms of after-tax income

Interpretation of FOCs (Kaplow 2008)

Suppose optimal income tax is in place

Perturbation of $T'(y)$ over interval $y + dy$ has following effects:

- Those in income interval $y + dy$ reduce ℓ since $T'(y)$ has risen
  - Loss of government revenue captured by $\epsilon w f(w)/(1+\epsilon)$
  - No change in their utility since marginal
- Those with income $< y$ not affected
- For those with income $> y$, $T'(y)$ unchanged, but pay increment more in taxes
  - No change in labor supply (quasilinearity)
  - There are $1 - F(w)$ of them, each paying an increment more in taxes
  - Term $B(w)$ is per capita value of the transfer in tax revenues from them to the government
- $B(w)$ rises with w, $C(w)$ may fall

Further Interpretation of FOCs (Saez 2001)

$H(y) =$ distribution of households by y (endogenous),
with density $h(y) = H'(y)$

Utility:

$$v(c, y) = c - \frac{\left(\frac{y}{w}\right)^{1+1/e}}{1 + 1/\epsilon} \quad \Rightarrow \quad y = \left(1 - T'(y)\right)^{\epsilon} w^{\epsilon+1}$$

Earnings elasticity: $\qquad \epsilon = \dfrac{dy}{d\left(1 - T'(y)\right)} \dfrac{1 - T'(y)}{y}$

Suppose optimal income tax is in place

Let $G(y)$ be average social value of giving one yen to all persons with income $> y$ (decreasing in y)

Increase $T'(y)$ by dT' over the interval $y + \Delta y \qquad \Longrightarrow$

Consequences of Tax Perturbation

- For those $y' > y$, tax liabilities rise by $dT'dy$, increase in government revenue: $dM = (1 - H(y))dT'dy$
- Loss in social welfare for those with $y' > y$ is $dW = -G(y)dM$
- For those in $y + dy$, $dy = -\epsilon y dT'/(1 - T')$, so tax revenue changes by $T'(y)dy$ (no loss of welfare); reduction in government revenue is

$$dB = -\frac{\epsilon y dT'}{1 - T'} h(y) T' dy$$

- In an optimum, $dM + dW + dB = 0$, leading to:

$$\frac{T'(y)}{1 - T'(y)} = \frac{1}{\epsilon} \cdot \frac{1 - H(y)}{yh(y)} \cdot (1 - G(y))$$

Similar interpretation as above

General Properties of Solution

- Marginal tax rate at the top zero (if distribution bounded)
- Marginal tax rate at the bottom zero unless bunching
- Marginal tax rate positive (and less that unity) in interior
- May be bunching at bottom (SOIC binding): marginal tax rate positive at end of bunching
- Simulations show inverted u-shape marginal tax profile, fairly flat in interior
- With quasilinear preferences and unbounded skill distribution, may be range of U-shaped marginal tax rates at upper end (Diamond)
- With maxi-min SWF, marginal tax rate positive at bottom and decreasing, average tax rates single-peaked $\Longrightarrow$
- Extension to extensive margin (participation): negative marginal tax rates at bottom $\Longrightarrow$

Maximin Case

Government problem: max $u(\underline{w})$ subject to
budget: $\int \big(y - c(y,u)\big) f(w) dw = 0$ and
Incentive constraint: $\dot{u} = h'(\ell)\ell / h(\ell)$.

Equivalent to max tax revenue $\int \big(y - c(y,u)\big) f(w) dw$ s.t.
$u(\underline{w}) \geq \overline{u}$ and incentive constraint

Since $W' = 0$ for $w > \underline{w}$, solution $\Longrightarrow$

$$\frac{T'}{1 - T'} = u' \int_{w}^{\overline{w}} \frac{dF(\tilde{w})}{u'(\tilde{w})} \cdot \frac{1 + \ell h''/h'}{wf}$$

$$\Longrightarrow T'(\underline{w}) > 0, \quad T'(w) \geq 0$$

Constant elasticity of labor supply case:

$$\frac{T'}{1 - T'} = u' \int_{w}^{\overline{w}} \frac{dF(\tilde{w})}{u'(\tilde{w})} \cdot \frac{1 + \epsilon^{-1}}{wf}$$

Interpretation

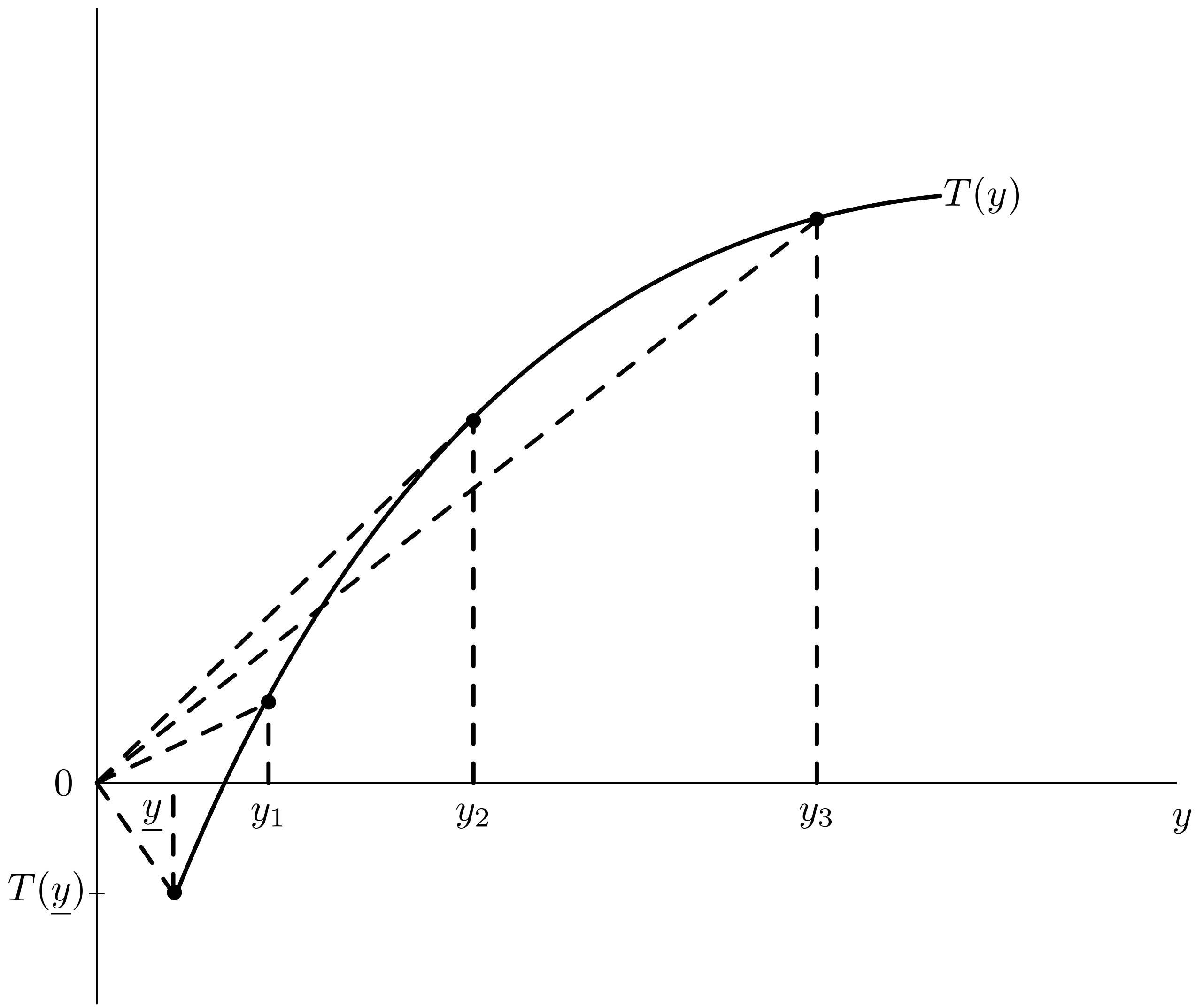
Assume single-peaked $f(w)$ and constant elasticity ℓ

- For w below the mode, T' is decreasing
- For w above the mode, T' is decreasing if wf is non-increasing
- If T' non-increasing in w, SOIC satisfied. Proof: from household FOC:

$$\frac{u'(c)}{h'(y/w)} = \frac{1}{(1-T')w} \quad \text{so} \quad \dot{y}, \dot{c} > 0$$

- Since $dT'(y(w))/dw = T''(y(w))\dot{y}(w)$, $T'' < 0$, so $T(y)$ increasing and strictly concave
- So, if $T(y(\underline{w})) < 0$, average tax rate $T(y)/y$ single-peaked

(FIGURE)



$T(y)$
0
$\underline{y}$
y_1
y_2
y_3
y
$T(\underline{y})$

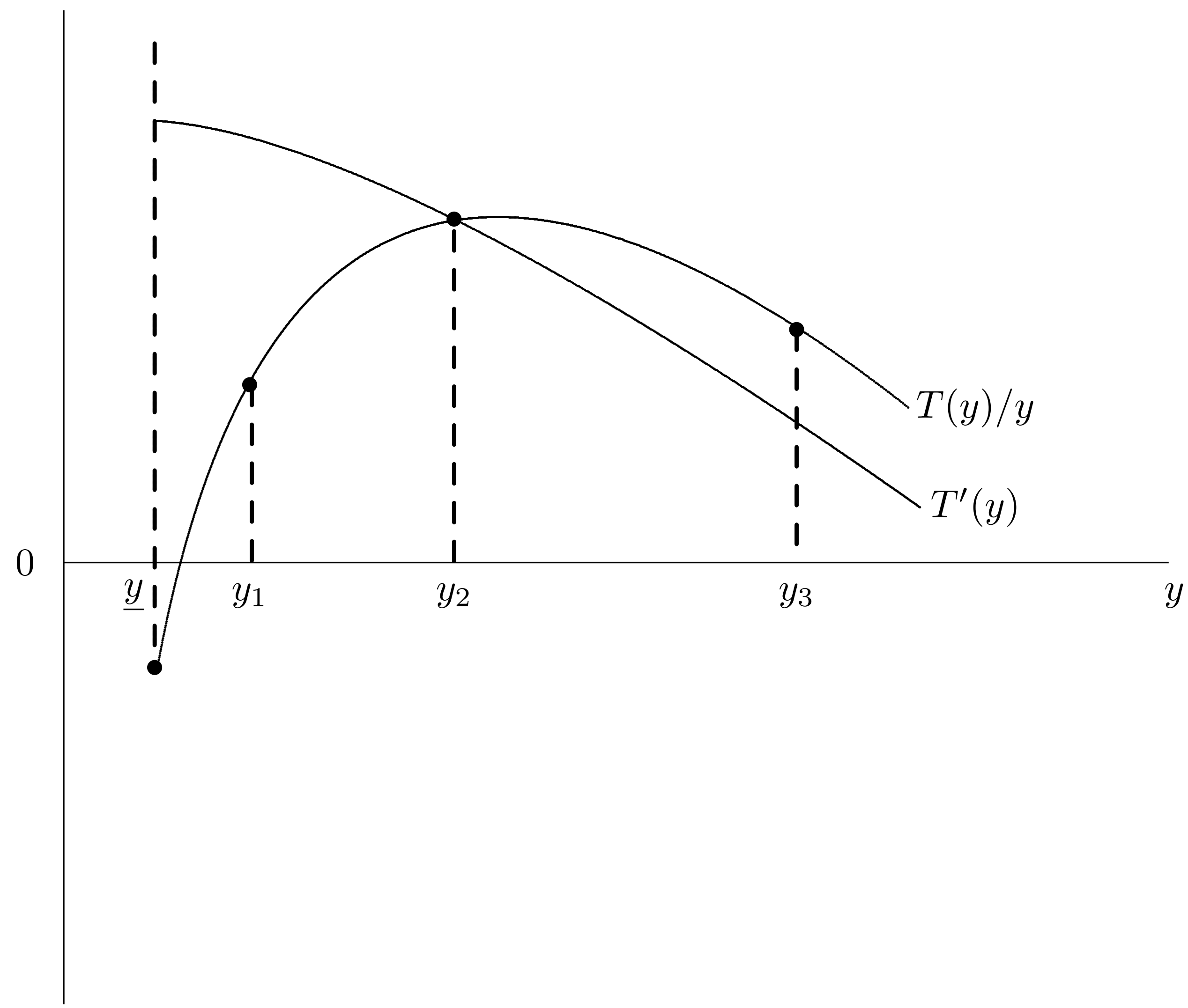


Figure Average and marginal tax rates

Labor Supply Choices along Extensive Margin

- Work involves fixed hours; households can choose occupation and/or participation (Saez 2002; Diamond 1980)
- $i = 0, \cdots, I$ occupations with fixed earnings y_i such that $y_i > y_{i-1}$, $y_0 = 0$
- h_i = proportion of population choosing i, $\sum h_i = 1$
- $c_i = y_i - T_i$, where tax $T_i \gtreqless 0$
- Occupation i labor supply function: $h_i(c_0, c_1, \cdots, c_I)$ (taste for leisure variable)
- The government's budget: $\sum_{i=0}^{I} h_i(c_0, c_1, \cdots, c_I) T_i = R$
- g_i = marginal social welfare weight for persons in i, $g_i > g_{i+1}$ for $i \geq 1$, $g_0 \gtreqless g_1$ (lazy vs. disabled)
- Utility quasilinear in c: $\Rightarrow \sum h_i(c_0, c_1, \cdots, c_I) g_i = 1$ (Marginal yen of government revenue valued as much as additional yen distributed to all income classes: unit increase in income of all persons leaves h_i unchanged)

Participation Choice

Labor supply into i: $h_i(c_i - c_0)$, with $h_i'(\cdot) > 0$, and elasticity
$\eta_i = (c_i - c_0)h_i'(\cdot)/h_i(\cdot)$

Let $\tau(y_i) = (T_i - T_0)/y_i$ (participation tax rate)
Then, optimal tax system satisfies:

$$\frac{T_i - T_0}{c_i - c_0} = \frac{\tau(y_i)}{1 - \tau(y_i)} = \frac{1 - g_i}{\eta_i} = \frac{\text{equity effect}}{\text{efficiency effect}} \quad \text{for} \quad i \geq 1$$

Proof: Increase dT_i causes
a) Direct Revenue Effect $= h_i dT_i$ with value $(1 - g_i)h_i dT_i$, and
b) Behavioral Effect $= dh_i = -h_i\eta_i dT_i/(c_i - c_0)$ so tax loss
$(T_i - T_0)dh_i$
At optimum, two effects sum to zero, leading to result

Implication:
1) If $g_0 > g_1 > g_2 \cdots$, for some $i < i^*$, $g_i > 1$, so $T_i < T_0 < 0$
for$i < i^* \implies$ MTR < 0 at bottom (EITC)

2) Maximin: $i^* = 0$, so $g_i = 0$ for $i > 0$, so $T_i > T_0$

Formal Treatment of Optimal Participation Tax

- Utility quasilinear in consumption
- Utility if working: $c_i = y_i - T_i$
- Utility if not working: $c_0 = -T_0 + \tilde{m}_i$
- Value of leisure $\tilde{m}_i$ distributed by $\Gamma_i(m_i)$
- Marginal type-i participant: $y_i - T_i = -T_0 + \hat{m}_i$
- Number of type-i participants:
  $n_i \Gamma_i(\hat{m}_i) = n_i \Gamma_i\big(y_i - T_i + T_0\big) \equiv h_i(\cdot)$
- Number of non-participants:
  $1 - \sum_{i \geq 1} n_i \Gamma_i\big(y_i - T_i + T_0\big) \equiv h_0$

Government Problem

$$\mathcal{L} = \sum_{i>0} h_i(y_i - T_i + T_0)u\big(y_i - T_i\big) + \sum_{i\geq 0}\int_{\hat{m}_i} u\big(-T_0 + m_i\big)d\Gamma_i(m_i)$$

$$+\lambda\Bigg(\sum_{i>0} h_i(y_i - T_i + T_0)T_i + \Big(1 - \sum_{i>0} h_i(y_i - T_i + T_0)\Big)T_0\Bigg)$$

FOCs with respect to T_i and T_0:

$-h_i u_i' + \lambda\big(h_i - (T_i - T_0)h_i'\big) = 0, \quad$ for $i > 0$

$-\sum_{i\geq 0}\int_{\hat{m}_i} u_{i0}' d\Gamma_i(m_i) + \lambda\big(h_0 + \sum_{i>0}(T_i - T_0)h_i'\big) = 0$

Define $g_i \equiv u_i'/\lambda$, $g_0 \equiv \sum_{i\geq 0}\int_{\hat{m}_i} u_{i0}' d\Gamma_i/(h_0\lambda)$

Then, these first-order conditions reduce to:

$$\frac{T_i - T_0}{c_i - c_0} = \frac{1 - g_i}{\eta_i}\ (i > 0), \quad \text{and} \quad \sum_{i\geq 0} h_i g_i = 1$$

Occupational Choice

Type$-i$ can opt for occupation $i-1$ and earn y_{i-1} instead of y_i

Labor supply to occupation i is $h_i(c_{i+1}-c_i, c_i-c_{i-1})$ with
$\epsilon_i = (c_i - c_{i-1})/h_i \cdot \partial h_i(\cdot)/\partial(c_i - c_{i-1})$

Optimal tax system satisfies, for all $i \geq 1$:

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\epsilon_i}\left[\frac{(1-g_i)h_i + (1-g_{i+1})h_{i+1} + \cdots + (1-g_I)h_I}{h_i}\right]$$

(Proof involves welfare effects of dT for occupations $i, i+1, \cdots, I$)

Implications

- Since $\sum h_i g_i = 1 = \sum h_i$, for $i > 0$ we have:
  $(1-g_i)h_i + (1-g_{i+1})h_{i+1} + \cdots + (1-g_I)h_I > 0$,
  so $T_i > T_{i-1}$ (MTRs > 0)
- At bottom and top,

$$\frac{T_1 - T_0}{c_1 - c_0} = \frac{1}{\epsilon_1}\cdot\frac{(g_0-1)h_0}{h_1} > 0, \quad \frac{T_I - T_{I-1}}{c_I - c_{I-1}} = \frac{1}{\epsilon_I}\cdot\frac{(1-g_I)h_I}{h_1} > 0$$

Both Participation and Occupational Choice

Supply in occupation $i = h_i(c_i - c_0, c_{i+1} - c_i, c_i - c_{i-1})$

- First argument due to participation
- Last two due to occupational choice

Optimal tax systems satisfies, for $i \geq 1$:

$$\frac{T_i - T_{i-1}}{c_i - c_{i-1}} = \frac{1}{\epsilon_i h_i} \sum_{j=1}^{I} h_j \left[1 - g_j - \eta_j \frac{T_j - T_0}{c_j - c_0} \right]$$

Comparing this with occupational choice only, g_j is replaced by

$$g_j + \eta_j \frac{T_j - T_0}{c_j - c_0}$$

If η large relative to ϵ, can be negative MTR at bottom (EITC vs. NIT)

Atkinson-Stiglitz Theorem: Continuous-Wage Case

- Return to intensive-margin setting
- Goods $x_i,\ i = 1, \cdots, n$ and labour ℓ
- Utility $u = u\big(x_1, \cdots, x_n, \ell\big)$
- Invert utility to obtain $x_1 = x_1(x_2, \cdots, x_n, \ell, u)$

Government can choose quantities of goods and labor subject only to resource and incentive constraints

$\Rightarrow$ Fully nonlinear income & commodity tax system

Government problem:

$\max \int W(u) f(w) dw$ subject to

$\int (w\ell - \sum x_i) f(w) dw = R$, and

$\dot{u} = -\ell u_\ell / w$

Control variables $x_2, \cdots, x_n, \ell$ with $x_1(x_2, \cdots, x_n, \ell, u)$

State variable u (all variables vary continuously with w)

Solution

Hamiltonian function is:

$\mathcal{H} = \big(W(u) + \lambda(w\ell - \sum x_i - R)\big) f(w) - \zeta \ell u_\ell / w$

FOC with respect to x_k:

$$-\lambda \left(1 + \frac{\partial x_1}{\partial x_k}\bigg|_u\right) f(w) - \zeta \frac{\ell}{w}\left(u_{\ell k} + u_{\ell 1}\frac{\partial x_1}{\partial x_k}\bigg|_u\right) = 0$$

Implementation

Nonlinear tax functions: $T(w\ell)$ and $t_i(x_i(w))$, with $t_1(x_1) = 0$

From household utility maximization (suppress type w):

$$\frac{\partial x_1}{\partial x_k}\bigg|_u = -\frac{u_k}{u_1} = -(1 + t_k')$$

where $t_k'\big(x_k(w)\big)$ is type-w's marginal tax rate on x_k

$\Longrightarrow$

Interpretation

$$t_k' = \frac{\zeta \ell u_k}{\lambda w f}\left(\frac{u_{\ell k}}{u_k} - \frac{u_{\ell 1}}{u_1}\right)$$

or

$$\frac{t_k'}{q_k} = \frac{\zeta \ell \alpha}{\lambda w f}\left(\frac{d\mathrm{Log}(u_k/u_1)}{d\ell}\right)$$

Therefore,

- If $u = u(f(x_1, \cdots, x_n), \ell)$, then $t_k'/q_k = 0$ for all $k = 1, \cdots, n$ (A-S Theorem)
- $t_k'/q_k \ > \ 0$ iff x_k more complementary with leisure than x_1
- Konishi-Laroque-Kaplow: If $u = u(f(x_1, \cdots, x_n), \ell)$, starting from any nonlinear income tax and $t_i \neq t_j$, move to uniform commodity taxes and adjust income tax: Pareto-improving
- Note: This yields different tax rates for different persons, which can only be implemented by nonlinear commodity taxes; two-type case addresses this $\Longrightarrow$

The Direct-Indirect Tax Mix: Two-Type Case

The Setting

- Utility: $v^i(x, z, y) \equiv u(x, z, y/w_i)$, where x, z and leisure $h - \ell$ are normal
- Government observes $y = w\ell$, not x and z
- Non-linear tax on y, indirect tax $t \gtreqless 0$ on z
- Consumption $c \equiv y - T(y) = x + (1 + t)z = x + qz$

Households: Max $v^i(x, z, y)$ s.t. $x + qz = c = y - T(y)$

Disaggregate into two stages

1. Choose y, c
2. Allocate c to x and z

Stage 2: Choice of Consumption Bundle

Types 1 and 2

Given c_i, y_i, household i max_z $v^i(c_i - qz_i, z_i, y_i)$

$\Longrightarrow$ Demand $z_i(q, c_i, y_i)$, $\partial z_i/\partial y_i \gtreqless 0$

$\Longrightarrow$ Indirect utility $w^i(q, c_i, y_i)$ with
$w^i_q = -z_i v^i_x, w^i_c = v^i_x, w^i_y = v^i_y$

$\Longrightarrow$ Single crossing: $-w^1_y/w^1_c > -w^2_y/w^2_c$

Mimicker: $max_{\hat{z}}$ $\hat{v}^2(c_1 - q\hat{z}_2, \hat{z}_2, y_1)$

$\Longrightarrow$ $\hat{z}_2(q, c_1, y_1)$, $\hat{w}^2(q, c_1, y_1)$

$\Longrightarrow$ $\hat{z}_2 > z_1$ if z more complementary with leisure than x, and vice versa

Stage 1: Choice of Labor Supply

Anticipating Stage 2, households choose c, y to maximize $w^i(q, c_i, y_i)$ s.t. $c_i = y_i - T(y_i)$

As above, we solve directly by letting government choose c_1, y_1, c_2, y_2 and t subject to budget and IC constraint

Households choose most preferred bundle (c_i, y_i)

Government Policy

Disaggregate into two stages:

1. Choice of an optimal non-linear income tax, given t
2. Welfare effect of changing t

Optimal Non-Linear Income Tax

Government problem, given t:

$$\max_{\{c_i, y_i\}} \quad w^1(q, c_1, y_1) + \rho w^2(q, c_2, y_2)$$

subject to

$$w^2(q, c_2, y_2) \geq \widehat{w}^2(q, c_1, y_1) \qquad (\gamma)$$

$$n_1(y_1 - c_1 + tz_1) + n_2(y_2 - c_2 + tz_2) = 0 \qquad (\lambda)$$

FOCs yield

$$-\frac{w_y^2}{w_c^2} = \frac{1 + t\partial z_2/\partial y_2}{1 - t\partial z_2/\partial c_2}$$

$\Longrightarrow -w_y^2/w_c^2 \neq 1$ (marginal tax rate at the top $\neq 0$)

Marginal tax rate at the bottom still positive

Denote Maximum Value Function for this problem by $W(t)$

Indirect Tax Perturbations

Envelope theorem: $\partial W/\partial t = \partial \mathcal{L}/\partial t$

Using FOCs from optimal income tax problem:

$$\frac{\partial W}{\partial t} = \gamma \widehat{v}_x^2(\widehat{z}_2 - z_1) + \lambda t \left(n_1 \frac{\partial \widetilde{z}_1}{\partial q} + n_2 \frac{\partial \widetilde{z}_2}{\partial q} \right)$$

where $\widetilde{z}_i$ is the compensated for demand for z_i

at $t = 0$: $$\left.\frac{\partial W}{\partial t}\right|_{t=0} > 0 \text{ if } \widehat{z}_2 > z_1$$

$\implies$ $t > 0$ if z is more complementary with leisure than is x

$\implies$ *Atkinson-Stiglitz Theorem:* $t = 0$ if
$u(x, z, G) = u(f(x, z), \ell)$ (weak separability)

Results generalize to many goods and many ability-types

Optimal Indirect Tax

Choose to such that $\partial \mathcal{W}/\partial t = 0$:

$$t^* = -\frac{\gamma \widehat{v}_x^2(\widehat{z}_2 - z_1)}{\lambda t \left(n_1 \partial \widetilde{z}_1/\partial q + n_2 \partial \widetilde{z}_2/\partial q \right)}$$

(Edwards-Keen-Tuomala result)

- Denominator is an efficiency or deadweight loss term
- Numerator is a redistributive effect due to relaxing IC constraint

Intuition

- Suppose z and leisure are complements so $\widehat{z}_2 > z_1$
- Start at $t = 0$, change $dt > 0$, adjust $dT_i = -z_i dt$ for $(i = 1, 2)$
- Then, $dw^1 = dw^2 = 0$, budget balances, and $d\widehat{w}^2, 0$, so IC relaxed
- Increase t until value of relaxing IC constraint just offset by marginal deadweight loss

Public Goods Provision: Marginal Cost of Public Funds

Utility: $v^i(c, y, G) \equiv u(c, y/w_i, G)$ where

$$\frac{v^i_G}{v^i_c} = \frac{u^i_G}{u^i_c} = MRS^i_{Gc}$$

Government problem:

$$\max_{\{c_i, y_i, G\}} \quad v^1(c_1, y_1, G) + \rho[v^2(c_2, y_2, G) - \overline{v}^2] \quad \text{s.t.}$$

$$v^2(c_2, y_2, G) \geq \widehat{v}^2(c_1, y_1, G) \qquad (\gamma)$$

$$n_1(y_1 - c_1) + n_2(y_2 - c_2) = pG \qquad (\lambda)$$

First-order conditions: (2)–(5) plus

$$v^1_G + \rho v^2_G + \gamma v^2_G - \gamma \widehat{v}^2_G - \lambda p = 0 \qquad (6)$$

Interpretation

Optimal income tax structure unchanged

Substitute (2) and (4) into (6):

$$\textit{Modified Samuelson}: \qquad n_1 \frac{v_G^1}{v_c^1} + n_2 \frac{v_G^2}{v_c^2} = p + \frac{\gamma \widehat{v}_c^2}{\lambda} \left[\frac{\widehat{v}_G^2}{\widehat{v}_c^2} - \frac{v_G^1}{v_c^1} \right]$$

$\Longrightarrow \sum MRS_{Gc} > MRT_{Gc}$ if $\widehat{MRS}^2_{Gc} > MRS^1_{Gc}$, and vice versa.

Intuition: Assume $\widehat{MRS}^2_{Gc} < MRS^1_{Gc}$

- Start at $\sum MRS_{Gc} = MRT_{Gc}$
- Change $dG > 0$ with $dT_1 = MRS^1_{Gc}$ and $dT_2 = MRS^2_{Gc}$

$\Longrightarrow dv^1 = dv^2 = 0$, budget balances, $d\widehat{v}^2 < 0$ so IC relaxed.

Further Comments

- MCPF is less than 1 if $\widehat{MRS}^2_{GC} < MRS^1_{GC}$, and vice versa
- Samuelson Rule applies if $u(c, \ell, G) = u(f(c, G), \ell)$ (weak separability in ℓ)
- *Reason:* Type 1 and mimicker have same c, G but $\ell_1 > \widehat{\ell}^2$
- If labor is more complementary with G than with c, the MCPF< 1
- *Reason:* Higher ℓ entails higher MRS_{Gc}, so $\widehat{MRS}^2_{Gc} < MRS^1_{Gc}$
- Kaplow: Even if income tax non-optimal, if preferences weakly separable, Samuelson condition should be satisfied if changes in G can be accompanied by adjustment in income tax liabilities

Environmental Externality: Pigouvian Taxation

Suppose z is now a dirty good

Utility: $u_x(x) + u_z(z) - y/w + e$ (quasilinear)
where $e = \overline{e} - \delta(n_1 z_1 + n_2 z_2)$ (externality)
$\delta =$ marginal damage

Nonlinear tax on y, excise tax t on z as before

Household choice of x, z, given c, y

$\text{Max}_{\{z\}} u_x(c - qz) + u_z(z) - y/w + e \;\Rightarrow\; z(q, c)$

$\Rightarrow\; w(q, c, y) + e,\; w_q = -zu'_x, w_c = u'_x, w_y = 1/w$

For mimicker: $\widehat{z}^2(q, c_1),\; \widehat{w}^2(q, c_1, y_1) + e$

Note: Given separability, $\widehat{z}^2 = z_1$

First-Best Government Policy

$$Max_{\{c_i, y_i, t\}} \; \rho_1 n_1 w^1(q, c_1, y_1) + \rho_2 n_2 w^2(q, c_2, y_2) + \overline{n}e$$

s.t.

$$n_1\big(y_1 - c_1 + tz_1(q, c_1)\big) + n_2\big(y_2 - c_2 + tz_2(q, c_2)\big) = R$$

where $\overline{n} = \rho_1 n_1 + \rho_2 n_2$

FOCs yield:

$$\lambda^o = \rho_1 w_y^1 = \rho_2 w_y^2$$

$$t^o = \frac{n_1 \delta}{w_y^1} + \frac{n_2 \delta}{w_y^2}$$

$\Rightarrow$ Equality of marginal social utility of incomes, and

$\Rightarrow$ Pigouvian tax equals sum of marginal damages evaluated by households (no social welfare weights)

Second-Best Government Policy

$$Max_{\{c_i, y_i, t\}} \; \rho_1 n_1 w^1(q, c_1, y_1) + \rho_2 n_2 w^2(q, c_2, y_2) + \overline{n} e$$

s.t. $w^2(q, c_2, y_2) \geq \widehat{w}^2(q, c_1, y_1)$

$$n_1\big(y_1 - c_1 + t z_1(q, c_1)\big) + n_2\big(y_2 - c_2 + t z_2(q, c_2)\big) = R$$

From the FOCs, we obtain, using $\widehat{z}^2 = z_1$:

$$t = \frac{\overline{n}\delta}{\lambda} = \frac{(\rho_1 n_1 + \rho_2 n_2)\delta}{\lambda}$$

$\Rightarrow$ Pigouvian tax equals sum of marginal social damages (using social weights ρ_1, ρ_2) in terms of government revenue λ where

$$\lambda = \frac{n_1 \rho_1 w_y^1 + n_2 \rho_2 w_y^2}{n_1 + n_2}$$

Interpretation of Pigouvian Tax

Rewrite Pigouvian tax as:

$$t = \frac{n_1 \delta}{\lambda / \rho_1} + \frac{n_2 \delta}{\lambda / \rho_2}$$

Since $\rho_1 w_y^1 > \lambda > \rho_2 w_y^2$ (marginal social utilities of income),

$$\frac{\lambda}{\rho_1} < w_y^1, \quad \frac{\lambda}{\rho_2} > w_y^2$$

⇒ Pigouvian tax puts more weight on marginal damages to low-wage persons than high-wage persons (Sandmo)

⇒ Pigouvian tax plays some redistributive role

Note: The assumptions of Atkinson-Stiglitz otherwise apply here

Time-Using Consumption

Two illustrative cases:

1. Consumption time a perfect substitute for leisure
2. Consumption time a perfect substitute for labour

Standard results apply in former case, including Atkinson-Stiglitz Theorem

Focus on latter case

Consumption Time a Labour Substitute

Utility: $u(x, z, x_0) = u(x, z, h - \ell - a_x x - a_z z)$
with $\ell = y/w$ and $c = x + qz$

Stage 2 problem of household i, given c_i, y_i, is:
$\text{Max}_{z_i} \; v^i(c_i - qz_i, z_i, h - y_i/w_i - a_x(c_i - qz_i) - a_z z_i)$
$\Rightarrow z_i(q, c_i, y_i), w^i(q, c_i, y_i)$
with $w^i_c = v^i_x - v^i_0 a_x, w^i_q = -w^i_c z_i, w^i_y = -v^i_0$
Similarly for mimicker $\widehat{z}_2(q, c_1, y_1), \widehat{w}^2(q, c_1, y_1)$

Government maximizes $\rho_1 n_1 w^1(q, c_1, y_1) + \rho_2 n_2 w^2(q, c_2, y_2)$ s.t
$w^2(q, c_2, y_2) \geq \widehat{w}^2(q, c_1, y_1)$ and
$n_1\big(y_1 - c_1 + tz_1(q, c_1, y_1)\big) + n_2\big(y_2 - c_2 + tz_2(q, c_2, y_2)\big) = R$

using FOCs: $$\frac{\partial \mathcal{L}}{\partial t} = \gamma \widehat{w}^2_c(\widehat{z}_2 - z_1) + \lambda t \left(n_1 \frac{\partial \widetilde{z}_1}{\partial q} + n_2 \frac{\partial \widetilde{z}_2}{\partial q} \right)$$

Interpretation

As before,
$$\left.\frac{\partial \mathcal{L}}{\partial t}\right|_{t=0} \gtreqless 0 \quad \text{as} \quad \widehat{z}_2 \gtreqless z_1$$

Denote the slope of an indifference curve in goods space by:

$$\sigma(x, z.\ell) = -\left.\frac{dz}{dx}\right|_{du=0}$$

Then,
$$\frac{\partial \sigma}{\partial \ell} \gtreqless 0 \quad \Longleftrightarrow \quad \widehat{z}_2 \gtreqless z_1$$

Assume weak separability:
$u(f(x,z), x_0) = u(f(x,z), h - \ell - a_x x - a_z z)$ so:

$$\sigma(x, z, \ell) = \frac{u_f f_x - u_0 a_x}{u_f f_z - u_0 a_z}$$

Interpretation, cont'd

Differentiate $\sigma(x, z, \ell)$ with respect to ℓ:

$$\frac{\partial \sigma}{\partial \ell} = \frac{u_0 u_{f0} - u_f u_{00}}{(u_f f_z - u_0 a_z)^2}(a_z f_x - a_x f_z)$$

Since the first term on the rhs is positive if goods are normal,

$$\frac{\partial \sigma}{\partial \ell} \gtreqless 0 \quad \Longleftrightarrow \quad \frac{a_z}{a_x} \gtreqless \frac{f_z}{f_x}$$

Thus, $t > 0$ if $a_z/a_x > f_z/f_x$, i.e., if the relative intensity of time use by z versus x exceeds the rate at which z can be substituted for x in the goods' sub-utility function.

Relatively high time intensity of use causes a higher tax rate to be imposed on a commodity

Non-Market Labour

Assumptions

- Market labour ℓ_m, non-market labour ℓ_n
- Utility: $u\big(f(x,z),\ell_m,\ell_n\big)$
- Market income y_m observable

Case I: Household Production

- ℓ_n produces unobserved, non-marketed goods
- A-S Theorem applies: common c; differences in ℓ; x,z same
- Nonlinear income tax affected: both progressivity and possibly direction of incentive constraint

Case II: Informal Economy

- ℓ_n gives y_n to purchase x,z
- A-S Theorem violated: c differs for mimicker and type-1
- If y_n higher for type-2's, $\widehat{z}_2 > z_1$, so $t > 0$