Intertemporal Optimal Taxation: Outline

Capital Income Taxation with Linear and Nonlinear Taxation

1. **Representative Household**
   - Basic two-period model
   - Capital income taxation?
   - Time-consistent taxation
   - Time-inconsistent preferences
   - Bequests

2. **Heterogenous Households: Nonlinear Taxation**
   - Basic two-period model
   - Differences in discount rates
   - Varying ability over time
   - Uncertainty

Two-Period Optimal Commodity Taxation

- Present and future consumption: c_1, c_2
- Variable labor $\ell (= h - c_0)$ in present period
- Utility: $u(c_1, c_2, \ell)$
- Consumer prices: $1 + \theta_1, 1 + \theta_2, w$ $(p_1 = p_2 = 1)$
- Intertemporal budget constraint:

$$(1 + \theta_1)c_1 + \frac{(1 + \theta_2)c_2}{1 + r} = w\ell \quad \text{or,} \quad q_1 c_1 + q_2 c_2 = w\ell$$

where q_1, q_2 are present value consumer prices

Tax Equivalences

Proportional commodity tax $\theta_1 = \theta_2 = \theta$ equivalent to labor income tax t_w in present value terms
(though time profiles of revenues differ)

Budget constraint with an income tax at the rate θ_m:

$$c_1 + \frac{c_2}{1 + (1 - \theta_m)r} = (1 - \theta_m)w\ell$$

$\Longrightarrow$ Equivalent to θ_1, θ_2 with $\theta_2 > \theta_1$

Any θ_1, θ_2 can be replicated by

- Wage and capital income tax: t_w, t_r (dual income tax)
- Income and wage tax: t_m, t_w
- Income and value-added tax: t_m, θ

Optimal Two-Period Tax Structure

Given present value of government revenue

Three-commodity Ramsey tax applies:

$$\frac{\tau_1}{\tau_2} = \frac{\theta_1/(1+\theta_1)}{\theta_2/(1+\theta_2)} = \frac{\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{10}}{\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{20}}$$

$\Longrightarrow \tau_1 = \tau_2$ or $\theta_1 = \theta_2$ if $\varepsilon_{10} = \varepsilon_{20}$

$\Longrightarrow t_w$ optimal if $\varepsilon_{10} = \varepsilon_{20}$

$\Longrightarrow t_w$ and $t_r > 0$ if $\varepsilon_{20} < \varepsilon_{10}$
(c_2 more complementary with leisure than c_1)

Generally, $t_r \neq t_w$

Case for schedular taxation (dual income tax)

OLG Extension: Atkinson-Sandmo

- Young supply labor, consume and save; old consume
- Population grows at rate n
- If $r > n$, increase in K increases steady state welfare
- In absence of intergenerational transfers, if $r > n$, may be preferable to augment Corlett-Hague tax with further tax on capital to increase, given form of utility function (Atkinson-Sandmo, King)
- Mitigated by use of consumption vs wage tax (Summers), or debt policy/intergenerational transfers

Four-Commodity Case

Household utility: $u(c_1, c_2, \ell_1, \ell_2)$
Two-period budget constraint with all taxes:

$$(1+\theta_1)c_1 + \frac{(1+\theta_2)c_2}{1+(1-\theta_r)r} = (1-\theta_{w1})w_1\ell_1 + \frac{(1-\theta_{w2})w_2\ell_2}{1+(1-\theta_r)r}$$

(Tax rates on labor and consumption can vary over time)
Only 3 tax rates needed to control 3 relative prices
Example 1: Commodity taxes zero ($\theta_1 = \theta_2 = 0$):

$$c_1 + \frac{c_2}{1+(1-\theta_r)r} = (1-\theta_{w1})w_1\ell_1 + \frac{(1-\theta_{w2})w_2\ell_2}{1+(1-\theta_r)r}$$

Example 2: Wage taxes zero ($t_{w1} = t_{w2} = 0$):

$$(1+\theta_1)c_1 + \frac{(1+\theta_2)c_2}{1+(1-\theta_r)r} = w_1\ell_1 + \frac{w_2\ell_2}{1+(1-\theta_r)r}$$

Generally, need either $\theta_1 \neq \theta_2$ or $t_{w1} \neq t_{w2}$

Zero Capital Income Tax?

Case 1: $\theta_1 = \theta_2 = 0$. No need for t_r to tax c_1 relative to c_2 if:

Expenditure function implicitly separable:

$$e(A(q_1, q_2, u), (1 - \theta_{w1})w_1, (1 - \theta_{w1})w_2, u)$$

Example: $u(f(c_1, c_2), \ell_1, \ell_2)$ with $f(\cdot)$ homothetic

Case 2: $t_{w1} = t_{w2} = 0$. No need for t_r to tax $w_1\ell_1$ versus $w_2\ell_2$ if:

Expenditure function implicitly separable:

$$e(q_1, q_2, u, B(w_1, w_2), u))$$

Generally, either θ_i or t_{wi} must be time-varying

Suppose not $\Longrightarrow$

Chamley-Judd Zero-Capital Tax Case

Suppose

- Preferences are $u(c_1, \ell_1) + \beta u(c_2, \ell_2)$
- Wage rate is identical in both periods
- $\beta = 1/(1+r)$ (steady state)

$\Longrightarrow$ Optimal $t_r = 0$, $\theta_1 = \theta_2$, $t_{w1} = t_{w2}$

$\Longrightarrow$ $c_1 = c_2$ and $\ell_1 = \ell_2$ (Steady state)

Optimal for capital taxes to be zero in the long run in a representative-agent dynamic model

Proof of Zero t_r

All prices and taxes are in present value terms

Consumer prices:

$q_1 = 1, q_2 = p_2 + t_{c2}, \omega_1 = w_1 + t_{w1}, \omega_2 = w_2 + t_{w2}$

Household: Max $u(c_1, \ell_1) + \beta u(c_2, \ell_2)$ s.t. $c_1 + q_2 c_2 = \omega_1 \ell_1 + \omega_2 \ell_2$

FOCs c_1, c_2 : $\quad u_c^1 = \alpha, \qquad \beta u_c^2 = \alpha q_2$

FOCs ℓ_1, ℓ_2 : $\quad u_\ell^1 = -\alpha \omega_1 \qquad \beta u_\ell^2 = -\alpha \omega_2$

Government Lagrangian:

$$\mathcal{L} = u(c_1, \ell_1) + \beta u(c_2, \ell_2) + \lambda[w_1 \ell_1 + w_2 \ell_2 - c_1 - p_2 c_2 - R]$$
$$+\gamma[u_c^1 c_1 + \beta u_c^2 c_2 + u_\ell^1 \ell_1 + \beta u_\ell^2 \ell_2]$$

The first-order conditions are:

$$u_c^1 - \lambda + \gamma[u_c^1 + u_{cc}^1 c_1 + u_{\ell c}^1 \ell_1] = 0 \qquad (c_1)$$
$$\beta u_c^2 - \lambda p_2 + \gamma\beta[u_c^2 + u_{cc}^2 c_2 + u_{\ell c}^2 \ell_2] = 0 \qquad (c_2)$$
$$u_\ell^1 + \lambda w_1 + \gamma[u_\ell^1 + u_{c\ell}^1 c_1 + u_{\ell\ell}^1 \ell_1] = 0 \qquad (\ell_1)$$
$$\beta u_\ell^2 + \lambda w_2 + \gamma\beta[u_\ell^2 + u_{c\ell}^2 c_2 + u_{\ell\ell}^2 \ell_2] = 0 \qquad (\ell_2)$$

Proof of $t_r = 0$, continued

Since $p_2 = \beta(= 1/(1+r)$ and $w_2 = \beta w_1$, conditions (c_2) and (ℓ_2) become:

$$u_c^2 - \lambda + \gamma[u_c^2 + u_{cc}^2 c_2 + u_{\ell c}^2 \ell_2] = 0 \tag{c_2'}$$

$$u_\ell^2 + \lambda w_1 + \gamma[u_\ell^2 + u_{c\ell}^2 c_2 + u_{\ell\ell}^2 \ell_2] = 0 \tag{ℓ_2'}$$

(c_1), (c_2'), (ℓ_1) and (ℓ_2') satisfied if $c_1 = c_2$ and $\ell_1 = \ell_2$
So, $u_c^1 = u_c^2$ and $u_\ell^1 = u_\ell^2$

Using household FOCs:

$$\frac{u_c^2}{u_c^1} = \frac{q_2}{\beta} = 1 = \frac{p_2}{\beta}, \qquad \frac{u_\ell^2}{u_\ell^1} = \frac{\omega_2}{\beta\omega_1} = 1 = \frac{w_2}{\beta w_1}$$

$\Longrightarrow q_2 = p_2$, so no tax on capital income

$\Longrightarrow q_2/q_1 = w_2/w_1$, so labor taxes are same over time

Infinite-Horizon (Ramsey) Case

Note: In multi-period context, constant tax on capital equivalent to increasing tax on consumption over time (Bernheim): suggests a low capital tax rate, or a capital tax rate that varies over time

Utility; $u(x_0, \ell_0) + \sum_{t=1}^{\infty} \beta^t u(x_t, \ell_t)$

Taxes allowed on wages and capital

- Capital income tax $\longrightarrow 0$ in long run (Chamley-Judd)
- If $u(x, \ell) = x^{1-\sigma}/(1-\sigma) + v(\ell)$, capital tax zero for $t > 0$
- Assumes representative agent model: but Ricardian equivalence violates biology/anthropology (Bernheim-Bagwell)
- Assumes full commitment

Multi-Period OLG Model

Two-period life-cycle

- Zero-capital tax no longer generally applies unless
  - Steady state with no saving, or
  - Utility $u(x, \ell) = x^{1-\sigma}/(1-\sigma) + v(\ell)$
- Liquidity constraints favor capital taxes (Hubbard-Judd)
  - Reallocate tax liabilities to future periods
- Especially with wage uncertainty (Aiyagari)
  - Excessive precautionary saving
- Simulations suggest high capital income tax (Conesa-Kitao-Krueger)

Time-Consistent Taxation

The Problem

- Taxpayers take long-run and short-run decisions
- Long-run decisions, like saving, create asset income that is fixed in the future
- Short-run decisions, like labor supply, create income in the same period
- Second-best optimal tax policy is determined before long-run decisions are taken
- Second-best tax policies are generally time-inconsistent: even benevolent governments will choose to change tax policies after long-run decisions are undertaken
- If households anticipate such re-optimizing, the outcome will be inferior to the second-best
- Governments may implement policies up front to mitigate that problem

General Consequences of Inability to Commit

- Excessive capital taxation (Fischer)
- Samaritan's dilemma (Bruce-Waldman, Coate): Government unable to help those who have chosen not to help themselves
- Mitigated by various measures
  - Restriction to consumption taxation
  - Incentives for asset accumulation
  - Mandatory saving
  - Under-investment in tax enforcement
  - Social insurance
  - Training

Commodity Tax Case: An Illustrative Model

Based on Fischer 1981 *Rev Econ Dyn & Control* and Persson and Tabellini survey in *Handbook of Public Economics*

- Two periods, two goods (c_1, c_2) and labor in period 2 (ℓ)
- Quasilinear utility: $u(c_1) + c_2 + h(1-\ell)$
- Time endowment 1, wealth endowment 1
- Wage rate $= 1$, interest rate $= 0$
- Second-period taxes: t_k, t_ℓ on k, ℓ
- Fixed government revenue R

Consumer problem

$$\text{Max}_{\{c_1,\ell\}}\ u(c_1) + (1-t_\ell)\ell + (1-t_k)(1-c_1) + h(1-\ell)$$

$$\implies c_1(1-t_k),\ c_1'(1-t_k) < 0,\ k(1-t_k) = 1 - c_1(1-t_k)$$

$$\implies \ell(1-t_\ell),\ \ell'(1-t_\ell) > 0$$

Indirect utility: $v(t_k, t_\ell)$, with $v_{t_k} = -(1-c_1), v_{t_\ell} = -\ell$

Government Policy

$$\text{Max}_{\{t_k, t_\ell\}}\ v(t_k, t_\ell)\ \ \text{s.t.}\ \ t_\ell \ell(1-t_\ell) + t_k k(1-t_k) = R$$

Second-best tax: $\dfrac{t_\ell}{1-t_\ell} = \dfrac{\lambda - 1}{\lambda}\dfrac{1}{\eta_\ell} > 0,\ \dfrac{t_k}{1-t_k} = \dfrac{\lambda - 1}{\lambda}\dfrac{1}{\eta_k} > 0$

where $\eta_\ell = (1-t_\ell)\ell'/\ell$ and $\eta_k = (1-t_k)k'/k$

- Ex post, government will reoptimize by treating k as fixed and set t_k as high as possible (e.g. $t_k = 1$)
- Households anticipate this and reduce saving
- Time-consistent equilibrium is inferior to second-best
- Government may react by providing ex ante saving incentives
- Inability to commit may be responsible for high capital income and wealth tax rates in practice
- Widespread use of investment and savings incentives
- Same phenomenon applies to human capital investment, investment by firms and housing

Time-Inconsistent Preferences

The Case of Sin Taxes (O'Donoghue and Rabin)

Assumptions

- Households consume x_t, z_t in period $t \in [0, T]$
- Utility: $u_t = v(x_t, \rho) - c(x_{t-1}, \gamma) + z_t,\ c_x, v_{x\rho}, c_{x\gamma} > 0$
- Income m, producer prices unity
- Government imposes tax θ on x, returns lump-sum revenue a
- Per period decision utility: $u^*(x, z) = v(x, \rho) - \beta c(x, \gamma) + z$
- Experienced utility: $u^{**}(x, z) = v(x, \rho) - c(x, \gamma) + z$

Ideal Behaviour

Max $u^{**}(x, z)$ s.t. $x + z = m \quad \Longrightarrow$

$$v_x(x^{**}, \rho) - c_x(x^{**}, \gamma) - 1 = 0,\ z^{**} = m - x^{**}$$

Actual Behaviour

Max $u^*(x, z)$ s.t. $(1 + \theta)x + z = m + a \quad \Longrightarrow$

$$v_x(x^*(\theta), \rho) - \beta c_x(x^*(\theta), \gamma) = 1 + \theta$$
$$z^*(\theta, a) = m + a - (1 + \theta)x^*(\theta)$$

Optimal Sin Taxes

When $t = 0$: $x^*(0) \geq x^{**}(0)$ as $\beta \leq 1$

Identical households

Optimal tax: $\theta^{**} = (1 - \beta)c_x(x^{**})$

$\Longrightarrow$ Pigouvian tax on externality imposed on one's self

Heterogeneous households

1. If $\beta = 1$ for all households, $\theta^* = 0$
2. If $\beta < 1$ for all, $\theta^* > 0$, but first best not achieved due to heterogeneity in γ, ρ, β
3. If $\beta < 1$ for some, $\beta = 1$ for others, $\theta^* > 0$: second-order effect of small tax if $\beta = 1$, first-order effect if $\beta < 1$

Note: Should the government interfere with consumer behaviour in the first place? (Paternalism or not)

Bequests

Motives

- Voluntary I: Altruism
- Voluntary II: Joy of giving
- Involuntary: Unintended
- Strategic: Requited transfer

Efficient Taxation

- Externality of voluntary transfers (benefits to donors and donees): Pigouvian subsidy on bequests
- Taxation of involuntary transfers fully efficient

Equitable Taxation

- Voluntary & strategic transfers: tax donors and donees
- Double counting?
- Ricardian equivalence?
- Equality of opportunity arguments

Dynamic Optimal Nonlinear Taxation

The Basic Two-Period, Two-Type Case (Diamond)

- c_i^j = consumption in period j by type i $(i, j = 1, 2)$
- $\ell_i^1 = y_i^1 / w_i$ labour supply by type i in period 1 only
- Utility: $u(c_i^1) - h(\ell_i^1) + \beta u(c_i^2)$
- Lifetime tax schedule (gov. observes c_i^1, c_i^2, or s)

Government problem (full commitment assumed)

$$\max\ n_1 \left(u(c_1^1) - h\Big(\frac{y_1^1}{w_1}\Big) + \beta u(c_1^2) \right) + n_2 \left(u(c_2^1) - h\Big(\frac{y_2^1}{w_2}\Big) + \beta u(c_2^2) \right)$$

s.t.

$$n_1\Big(y_1^1 - c_1^1 - \frac{c_1^2}{1+r}\Big) + n_2\Big(y_2^1 - c_2^1 - \frac{c_2^2}{1+r}\Big) = R \qquad (\lambda)$$

$$u(c_2^1) - h\Big(\frac{y_2^1}{w_2}\Big) + \beta u(c_2^2) \geq u(c_1^1) - h\Big(\frac{y_1^1}{w_2}\Big) + \beta u(c_1^2) \qquad (\gamma)$$

Basic Case, cont'd

Focus is on capital income taxation

First-order conditions on consumption:

$$\begin{aligned}
c_1^1 &: \quad (n_1 - \gamma)u'(c_1^1) - \lambda n_1 = 0 \\
c_1^2 &: \quad (n_1 - \gamma)\beta u'(c_1^2) - \lambda n_1/(1+r) = 0 \\
c_2^1 &: \quad (n_2 + \gamma)u'(c_2^1) - \lambda n_2 = 0 \\
c_2^2 &: \quad (n_2 + \gamma)\beta u'(c_2^2) - \lambda n_2/(1+r) = 0
\end{aligned}$$

$$\Longrightarrow \qquad \frac{u'(c_1^1)}{\beta u'(c_1^2)} = \frac{u'(c_2^1)}{\beta u'(c_2^2)} = 1 + r$$

$\Longrightarrow$ No tax on capital income: A-S Theorem applies

Note: y_2^j conditions give zero marginal tax rate at the top

Extension 1: Different Utility Discount Rates

Suppose $\beta_1 \neq \beta_2$, so government objective becomes:

$$\max\ n_1 \left(u(c_1^1) - h\left(\frac{y_1^1}{w_1}\right) + \beta_1 u(c_1^2) \right) + n_2 \left(u(c_2^1) - h\left(\frac{y_2^1}{w_2}\right) + \beta_2 u(c_2^2) \right)$$

and the incentive constraint is:

$$u(c_2^1) - h\left(\frac{y_2^1}{w_2}\right) + \beta_2 u(c_2^2) \geq u(c_1^1) - h\left(\frac{y_1^1}{w_2}\right) + \beta_2 u(c_1^2)$$

Note: Utilitarian objective problematic with different preferences: may want different welfare weight on two types

First-order conditions yield

$$\frac{u'(c_2^1)}{\beta_2 u'(c_2^2)} = 1 + r = \frac{n_2 - \gamma}{n_2 - \gamma\beta_2/\beta_1} \frac{u'(c_1^1)}{\beta_1 u'(c_1^2)}$$

Different Utility Discount Rates, cont'd

Diamond argues $\beta_2 > \beta_1$ is plausible:

$$\Longrightarrow \qquad \frac{u'(c_1^1)}{\beta_1 u'(c_1^2)} \; < \; 1 + r \quad \text{if} \quad \beta_2 > \beta_1$$

$\Longrightarrow$ Implicit tax on savings of low-wage types

Intuition: Taxing savings of low-wage types reduces their second-period consumption, makes it more costly for high-wage types to mimic, given their lower utility discounting

With linear tax on savings (dual income tax), case for positive linear tax since high-wage types have higher savings rates

Extension 2: Earnings in Both Periods: Age-Dependent Taxation

- Wages in period j are w_i^j for $i, j = 1, 2$
- No uncertainty
- Identical preferences: $u(c^1) - h(\ell^1) + \beta u(c^2) - \beta h(\ell^2)$
- Government can commit to two-period tax system
- Fully nonlinear tax on present and future income
- Assume lifetime incentive constraint applies to type-2's

Government problem

$$\max \sum_{i=1,2} n_i \left(u(c_i^1) - h\left(\frac{y_i^1}{w_i}\right) + \beta u(c_i^2) - \beta h\left(\frac{y_i^2}{w_i}\right)\right)$$

$$\text{subject to } \sum_{i=1,2} n_i \left(y_i^1 + \frac{y_i^2}{1+r} - c_i^1 - \frac{c_i^2}{1+r}\right) = R \qquad (\lambda)$$

$$u(c_2^1) - h\left(\frac{y_2^1}{w_2}\right) + \beta u(c_2^2) - \beta h\left(\frac{y_2^2}{w_2}\right) \geq u(c_1^1) - h\left(\frac{y_1^1}{w_2}\right) + \beta u(c_1^2) - \beta h\left(\frac{y_1^2}{w_2}\right) \qquad (\gamma)$$

Tax Smoothing

The FOCs for c_i^j, y_i^j are:

$$(n_2+\gamma)u'(c_2^1) = \lambda n_2 = \frac{n_2+\gamma}{w_2^1} h'\left(\frac{y_2^1}{w_2^1}\right) \qquad (c_2^1, y_2^1)$$

$$(n_2+\gamma)\beta u'(c_2^2) = \frac{\lambda n_2}{1+r} = \frac{n_2+\gamma}{w_2^2} \beta h'\left(\frac{y_2^2}{w_2^2}\right) \qquad (c_2^2, y_2^2)$$

$$(n_1-\gamma)u'(c_1^1) = \lambda n_1 = \frac{n_1}{w_1^1} h'\left(\frac{y_1^1}{w_1^1}\right) - \frac{\gamma}{w_2^1} h'\left(\frac{y_1^1}{w_2^1}\right) \qquad (c_1^1, y_1^1)$$

$$(n_1-\gamma)\beta u'(c_1^2) = \frac{\lambda n_1}{1+r} = \frac{n_1}{w_1^2} \beta h'\left(\frac{y_1^2}{w_1^2}\right) - \frac{\gamma}{w_2^2} \beta h'\left(\frac{y_1^2}{w_2^2}\right) \quad (c_1^2, y_1^2)$$

$$\Longrightarrow \qquad \frac{u'(c_1^1)}{\beta u'(c_1^2)} = \frac{u'(c_2^1)}{\beta u'(c_2^2)} = 1+r$$

$\Longrightarrow$ No tax on savings

Tax Smoothing, cont'd

From conditions on c_2^j, y_2^j:

$$\frac{h'(y_2^1/w_2^1)}{u'(c_2^1)w_2^1} = 1 = \frac{h'(y_2^2/w_2^2)}{u'(c_2^2)w_2^2} \quad \Rightarrow \quad \text{Tax smoothing for 2's}$$

For type-1's, let $h'(\ell_i) = \ell_i^\sigma$; conditions on y_1^1, y_1^2 become:

$$\frac{h'(y_1^1/w_1^1)w_1^2}{h'(y_1^2/w_1^2)w_1^1}\Delta = \beta(1{+}r), \text{ where } \Delta = \left(\frac{w_2^2}{w_2^1}\right)^{\sigma+1} \frac{n_1 {w_2^1}^{\sigma+1} - \gamma {w_1^1}^{\sigma+1}}{n_1 {w_2^2}^{\sigma+1} - \gamma {w_1^2}^{\sigma+1}}$$

$\Rightarrow$ Tax smoothing for 1's if $\Delta = 1$: i.e., if $w_1^2/w_1^1 = w_2^2/w_2^1$ (identical age-earnings profiles, assuming $h'(\ell_i) = \ell_i^\sigma$)

$\Rightarrow$ If $w_1^2/w_1^1 < w_2^2/w_2^1$, marginal tax rate for 1's rises over time

Extension 3: Uncertain Future Wage Rates

- Two periods, 1 and 2
- Common wage w^1 in period 1, and either w_1^2 or w_2^2 in period 2
- Labor supply chosen after w revealed (incentive constraint in period 2 only)
- $n_i^2 =$ distribution of i's in period 2
- Expected utility: $u(c^1) - h(y^1/w^1) + \beta \sum_{i=1,2} n_i^2 \big(u(c_i^2) - h(y_i^2/w_i^2)\big)$

Government problem:

$$\max \; u(c^1) - h\left(\frac{y^1}{w^1}\right) + \beta \sum_{i=1,2} n_i^2 \left[u(c_i^2) - h\left(\frac{y_i^2}{w_i^2}\right)\right] \quad \text{s.t.}$$

$$y^1 - c^1 + (1+r)^{-1} \sum_{i=1,2} n_i^2 \left(y_i^2 - c_i^2\right) \geq G \qquad (\lambda)$$

$$u(c_2^2) - h\left(\frac{y_2^2}{w_2^2}\right) \geq u(c_1^2) - h\left(\frac{y_1^2}{w_2^2}\right) \qquad (\gamma)$$

Tax on Savings with Uncertain Wage Rates

FOCs on c^1, c_i^2: $\qquad u'(c^1) = \lambda$

$$\beta(n_1^2 - \gamma)u'(c_1^2) - \frac{\lambda n_1^2}{1+r} = 0$$

$$\beta(n_2^2 + \gamma)u'(c_2^2) = \frac{\lambda n_2^2}{1+r} = 0$$

$$\Longrightarrow \qquad u'(c^1) = \beta(1+r)\Big[\sum_i n_i^2 u'(c_i^2) - \gamma\big(u'(c_1^2) - u'(c_2^2)\big)\Big]$$

If incentive constraint binding, $c_2^2 > c_1^2$, so

$$\frac{u'(c^1)}{\beta \sum_i n_i^2 u'(c_i^2)} < 1 + r \quad \Longrightarrow \quad \text{Tax on savings}$$

(Reducing saving makes it harder for 2's to mimic 1's in period 2)

Uncertainty and Earnings Tax Progressivity

- Suppose labor supplied before uncertainty resolved
- Income tax progressivity affected
- Progressivity higher or lower with ex post vs ex ante uncertainty (Eaton-Rosen)$\Rightarrow$ Progressivity enhances insurance but reduces precautionary labor supply:
- Depends on balance between coefficient of risk aversion and coefficient of prudence (Low-Maldoom):

$$P(x) = -\frac{u'''(x)}{u''(x)} \Big/ \frac{u''(x)}{u'(x)} \quad : \qquad P(x) \uparrow \Rightarrow \text{Prog} \downarrow$$

- Social insurance may induce socially-beneficial risk-taking (Sinn): enhance case for progressivity
- To extent that risk is insurable, less needs to be done via income tax (Cremer-Pestieau)

Wage Uncertainty and Goods Taxation

Cremer and Gahvari (1995): Wage rates uncertain and some goods must purchased before wage rate revealed, other goods and labor supply must chosen after wage rate revealed

- Assume weak separability applies
- No differential tax on goods purchased ex post
- Lower tax on goods purchased ex ante: Makes it more difficult for ex post high-wage types to mimic low-wage types
- Provides justification for preferential treatment of housing and other consumer durables

Other Extensions

- Quantity controls: In-kind transfers
- Quantity controls: Workfare
- Price controls: Minimum wage
- Information acquisition: Tagging
- Information acquisition: Monitoring
- Multiple dimensions: Risk, family size
- Tax evasion, corruption, extortion
- Commitment issues
- Human capital accumulation
- Involuntary unemployment: search and unemployment insurance

Policy Implications from Optimal Tax Theory

- Atkinson-Stiglitz theorem: broad-based VAT
- Case for separate capital income tax: dual income tax
- Production efficiency and case for VAT
- Progressivity
- Extensive margin and low-wage subsidy
- Equality of opportunity: inheritance tax, targeted child transfers
- Behavioral economics and paternalistic taxation
- Behavioral economics and mandatory savings
- Political economy?