OPTIMAL TAXATION: LESSONS FOR TAX POLICY

Special Lectures at the University of Tokyo

International Program in Economics and

Center for International Research on the Japanese Economy

by

Robin Boadway, Queen's University, Canada

December 5–6, 2011

Purpose

To study the main results from the optimal tax literature that have been relevant for tax policy

Topic Outline

1. Optimal Linear Tax Analysis
2. Optimal Nonlinear Tax Analysis
3. Two-Period Analysis and Capital Income Taxation
4. Further Issues

1 Optimal Linear Tax Analysis

Representative household setting

- Efficient taxation: Ramsey Rule
- Corlett-Hague Theorem
- Conditions for uniform taxation
- Externalities

Heterogeneous households

- Social welfare functions
- First-best outcomes
- Optimal linear progressive taxation
- Commodity taxation: Deaton conditions
- Production Efficiency Theorem

2 Optimal Nonlinear Tax Analysis

Optimal nonlinear income taxation

- Two wage-type case (Stern-Stiglitz)
- Multiple discrete wage-types
- Continuous wage types (Mirrlees)
- Maximin case
- Extensive-margin labor supply

Income vs commodity taxation

- Atkinson-Stiglitz Theorem
- Generalization of Atkinson-Stiglitz Theorem
- Differential taxation of leisure complements

3 Two-Period Analysis and Capital Income Taxation

Representative household models

- The Corlett-Hague analogue
- Overlapping-generations case
- Time-consistent taxation
- Time-inconsistent preferences
- Bequests

Two wage-type nonlinear taxation

- Identical preferences
- Different discount factors
- Age-dependent taxation
- Uncertain future wage rates
- Liquidity constraints

4 Further Issues

- Human capital investment
- Uncertain earnings
- Marginal cost of public funds
- Time-using consumption
- Non-tax instruments: minimum wage, in-kind transfers, workfare
- Involuntary unemployment

Methodological Note: The Envelope Theorem

Consider the constrained maximization problem:

$\text{Max}_{\mathbf{x}} f(\mathbf{x};\mathbf{y})$ s.t. $g(\mathbf{x};\mathbf{y}) = 0$, where $\mathbf{x} =$ vector of choice variables and $\mathbf{y} =$ vector of exogenous variables

The Lagrange expression is $\mathcal{L} = f(\mathbf{x};\mathbf{y}) + \lambda g(\mathbf{x};\mathbf{y})$

The first-order conditions are

$$\frac{\partial f(\mathbf{x};\mathbf{y})}{\partial x_i} + \lambda \frac{\partial g(\mathbf{x};\mathbf{y})}{\partial x_i} = 0, \quad \forall i$$

The solution gives $\mathbf{x}(\mathbf{y})$, and a value function $F(\mathbf{y}) \equiv f\big(\mathbf{x}(\mathbf{y});\mathbf{y}\big)$
By the envelope theorem:

$$\frac{\partial F(\mathbf{y})}{\partial y_j} = \frac{\partial \mathcal{L}}{\partial y_j} = \frac{\partial f(\mathbf{y})}{\partial y_j} + \lambda \frac{\partial g(\mathbf{y})}{\partial y_j}$$

We use this frequently for both consumer maximization problems and social welfare maximization problems

Optimal Commodity Taxation: Identical Households

The Ramsey Problem

- $n+1$ commodities: $x_1, \cdots, x_n$ goods, $x_0 = h - \ell$ leisure
- Representative household utility: $u(x_1, \cdots, x_n, h-\ell)$
- Producer prices: $p_i, i = 1, \cdots, n$ and $p_0 = w = 1$
- Taxes: $t_i,\ i = 1, \cdots, n,\ t_0 = 0$
- Consumer prices: $q_i = p_i + t_i,\ i = 1, \cdots, n$ and $w = 1$

Ad valorem taxes:

$$\tau_i = \frac{t_i}{q_i}, \quad \theta_i = \frac{t_i}{p_i} \quad \implies \quad \tau_i = \frac{t_i}{p_i + t_i} = \frac{\theta_i}{1+\theta_i}, \quad \theta_i = \frac{\tau_i}{1-\tau_i}$$

Government is principal, households are agents

The Household Problem

$$\max_{\{x_i,\ell\}} \; u(x_1,\cdots,x_n,h-\ell) \qquad \text{s.t.} \qquad \sum_{i=1}^{n} q_i x_i - \ell = 0$$

Lagrangian expression:

$$\mathcal{L}(x_i,\ell,\alpha) = u(x_1,\cdots,x_n,h-\ell) - \alpha\left[\sum_{i=1}^{n} q_i x_i - \ell\right]$$

$\Longrightarrow$ uncompensated demands $\mathbf{x}(\mathbf{q},w), \ell(\mathbf{q},w)$

$\Longrightarrow$ indirect utility: $v(\mathbf{q},w) \equiv u\big(\mathbf{x}(\mathbf{q},w), \ell(\mathbf{q},w)\big)$

Envelope theorem:

$$\frac{\partial v}{\partial q_i} = -\alpha x_i(\mathbf{q},w), \; i=1,\cdots,n, \quad \frac{\partial v}{\partial w} = \alpha \ell(\mathbf{q},w), \quad \frac{\partial v}{\partial m} = \alpha$$

$\Longrightarrow$ α is marginal utility of household income

Production and Tax Normalizations

- Production possibilities are linear: $\sum_{i=1}^{n} p_i x_i = w\ell - R$ where $R =$ resources used by government
- Production constraint is homogeneous of degree zero in $\mathbf{p}$ and w: normalize producer prices by $w = 1$.
- Consumer demands are homogeneous of degree zero in $\mathbf{q}$ and w: normalize consumer prices by $w = 1 \Longrightarrow t_0 = 0$

Subtracting the production constraint from the consumer's budget constraint yields government budget constraint:

$$\sum_{i=1}^{n} t_i x_i = R$$

$\Longrightarrow$ One of household budget constraint, production constraint & government budget constraint are redundant

The Government's Problem

Lagrangian: $$\mathcal{L}(\mathbf{t},\lambda) = v(\mathbf{q}) + \lambda\left[\sum_{i=1}^{n} t_i x_i(\mathbf{q}) - R\right]$$

$\Longrightarrow$ λ is marginal utility of government revenue
$\Longrightarrow$ Production constraint redundant

First-order conditions:

$$t_k \;:\quad \frac{\partial v}{\partial q_k} + \lambda\left[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k}\right] = 0 \qquad k = 1,\cdots,n$$

$$\lambda \;:\quad \sum_{i=1}^{n} t_i x_i = R$$

$\Longrightarrow$ $t_k(R)$ and $\lambda(R)$
(No guarantee that second-order conditions satisfied)

Interpretation of Optimal Tax Rules

Rewrite FOC using envelope theorem, $\partial v / \partial q_i = -\alpha x_i(\mathbf{q}, w)$:

$$\sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) x_k \qquad k = 1, \cdots, n \tag{1}$$

Note that $\lambda > \alpha$:

- Marginal value of a yen to the government exceeds the marginal value of a yen to the household (since it is costly to transfer a yen from households to the government)

Using (1), various interpretations can be given to the optimal tax structure $\Longrightarrow$

The Inverse Elasticity Rule (Partial Equilibrium)

Assume preferences are quasilinear in leisure and additive:

$$u(x_1, \cdots, \ell) = u_1(x_1) + u_2(x_2) + \cdots + u_n(x_n) + (h - \ell)$$

$\Longrightarrow$ Demand functions for goods: $x_i(q_i),\ i = 1, \cdots, n$

Condition (1) then becomes:

$$t_k \frac{\partial x_k}{\partial q_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) x_k \qquad k = 1, \cdots, n$$

Interpretation 1: Inverse elasticity rule:

$$\tau_k = \frac{t_k}{q_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) \frac{1}{\eta_{kk}} \quad k = 1, \cdots, n$$

where $\eta_{kk} = (\partial x_k / \partial q_k)(q_k / x_k) < 0$ (elasticity of demand for x_k)

Interpretation 2: Proportional reduction approximation:

$$\frac{t_k}{x_k} \frac{\partial x_k}{\partial q_k} = \frac{\Delta q_k}{x_k} \frac{\partial x_k}{\partial q_k} \cong \frac{\Delta x_k}{x_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) \quad k = 1, \cdots, n$$

The Ramsey Rule

The Slutsky equation:

$$\frac{\partial x_i}{\partial q_k} = \left.\frac{\partial x_i}{\partial q_k}\right|_u - x_k \frac{\partial x_i}{\partial m} = s_{ik} - x_k \frac{\partial x_i}{\partial m} \quad i, k = 1, \cdots, n$$

Substitute Slutsky equation into (1):

$$\frac{\sum_i t_i s_{ik}}{x_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) + \sum_i t_i \frac{\partial x_i}{\partial m} = -(1-b) \quad k = 1, \cdots, n$$

where $b = \alpha/\lambda + \sum t_i \partial x_i/\partial m$, *net social marginal utility of income*

By symmetry of the substitution effect ($s_{ik} = s_{ki}$):

$$\frac{\sum_i t_i s_{ki}}{x_k} = -(1-b) \quad k = 1, \cdots, n \tag{2}$$

where $b < 1$ if $R > 0$: *Ramsey proportionate reduction rule*

Three-Commodity Case (Harberger)

The Ramsey rule (2) can be written

$$k = 1 \ : \qquad t_1 s_{11} + t_2 s_{12} = -(1-b)x_1$$

$$k = 2 \ : \qquad t_1 s_{21} + t_2 s_{22} = -(1-b)x_2$$

Eliminating $(1-b)$, we obtain:

$$\frac{t_1}{t_2} = \frac{s_{22}x_1 - s_{12}x_2}{s_{11}x_2 - s_{21}x_1} = \frac{s_{22}/x_2 - s_{12}/x_1}{s_{11}/x_1 - s_{21}/x_2}$$

Multiplying by q_2/q_1:

$$\frac{t_1/q_1}{t_2/q_2} = \frac{\tau_1}{\tau_2} = \frac{\varepsilon_{22} - \varepsilon_{12}}{\varepsilon_{11} - \varepsilon_{21}}$$

Compensated demands are homogeneous of degree zero, $\sum_i \varepsilon_{ji} = 0$, so

$$\frac{\tau_1}{\tau_2} = \frac{\varepsilon_{22} + \varepsilon_{11} + \varepsilon_{10}}{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{20}} \qquad (3)$$

Interpretation: Corlett-Hague Theorem

Since $\varepsilon_{11} + \varepsilon_{22} < 0$, (3) says that $\tau_1 > \tau_2$ if $\varepsilon_{10} < \varepsilon_{20}$
i.e., x_1 is relatively more complementary with leisure than x_2

Corlett-Hague Theorem: Impose a higher tax rate on the good that is more complementary with leisure

Intuition: Leisure is taxed indirectly by taxing its complement

Corollary: Taxes are proportional if $\varepsilon_{10} = \varepsilon_{20}$ (both goods equally complementary with leisure)

Proportional tax on goods equivalent to tax on labor income

Uniform Commodity Taxation

If $t_i/q_i = \tau$ for all goods, (2) gives:

$$\frac{\sum_i t_i s_{ki}}{x_k} = \tau \frac{\sum_i s_{ki} q_i}{x_k} = \tau \sum_{i=1}^{n} \varepsilon_{ki} = -(1-b) \qquad k = 1, \cdots, n$$

By homogeneity of compensated demands, $\sum_i \varepsilon_{ki} = 0$

$\Longrightarrow t_i/q_i = \tau$ if $\tau \varepsilon_{k0} = (1-b)$, $k = 1, \cdots, n$, or

$$\varepsilon_{k0} = \varepsilon_{j0} \qquad \forall j, k = 1, \cdots, n$$

All goods must be equally substitutable with leisure

Sufficient Condition for Uniform Taxation

Uniform taxation if: Expenditure functions are of form
$e(f(\mathbf{q}, u), w, u)$

Proof: The compensated demand for good i is given by

$$x_i(\mathbf{q}, w, u) = \frac{\partial e}{\partial f}\frac{\partial f}{\partial q_i}$$

Substitution effect with respect to w is:

$$s_{i0} = \frac{\partial x_i(\mathbf{q}, w, u)}{\partial w} = \frac{\partial^2 e}{\partial f \partial w}\frac{\partial f}{\partial q_i}$$

so,

$$\varepsilon_{i0} = \frac{s_{i0} w}{x_i} = w\frac{\partial^2 e}{\partial f \partial w}\left(\frac{\partial e}{\partial f}\right)^{-1} = \varepsilon_{k0}$$

Example: $u(f(x_1, \cdots, x_n), x_0)$, with $f(\cdot)$ homothetic

Intuition: Preferences for goods independent of x_0, income elasticities of demand all unity. Slutsky equation yields:

$$\frac{\partial x_i}{\partial q_k} = \frac{\partial x_k}{\partial q_i} \qquad \forall i, k = 1, \cdots, n$$

$$(1) \Longrightarrow \qquad \sum_{i=1}^{n} \frac{t_i \partial x_k / \partial q_i}{x_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) \qquad k = 1, \cdots, n$$

$$\text{or,} \quad \sum_{i=1}^{n} \frac{\Delta q_i \partial x_k / \partial q_i}{x_k} \cong \frac{\Delta x_k}{x_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) \qquad k = 1, \cdots, n$$

Proportional tax on ℓ reduces income and thus all goods demands proportionately since relative goods prices q_i have not changed

Environmental Taxation

Good x_1 affects quality of environment $e = f(Hx_1)$, $f' < 0, f'' \leq 0$

Separable utility: $u(x_0, \cdots, x_n, e) = u(h(x_0, \cdots, x_n), e)$

Household demands $x_i(q, w)$ and indirect utility $v(q, w, e)$

Government Lagrangian:

$$\mathcal{L}(t_i, \lambda) = Hv(q, w, f(Hx_1(q, w))) + \lambda \left[\sum_{i=1}^{n} Ht_i x_i(q, w) - R \right]$$

FOCs:

$$H \left[\frac{\partial v}{\partial q_k} + H \frac{\partial v}{\partial e} f' \frac{\partial x_1}{\partial q_k} \right] + \lambda \left[Hx_k + \sum_{i=1}^{n} Ht_i \frac{\partial x_i}{\partial q_k} \right] = 0 \qquad k = 1, \cdots, n$$

Using the envelope theorem and rearranging:

$$\alpha x_k = \lambda \left[x_k + \sum_{i=1}^{n} t_i \frac{\partial x_i}{\partial q_k} + H \frac{u_e}{\lambda} f' \frac{\partial x_1}{\partial q_k} \right] \qquad k = 1, \cdots, n$$

Interpretation

Define 'shadow taxes' net of Pigouvian tax:

$$t_1^* = t_1 + H\frac{u_e}{\lambda}f' = t_1 - t_P, \qquad t_i^* = t_i, \;\; i = 2, \cdots, n$$

FOCs: $$\sum_{i=1}^{n} t_i^* \frac{\partial x_i}{\partial q_k} = -\left(\frac{\lambda - \alpha}{\lambda}\right) x_k \qquad k = 1, \cdots, n$$

(Analogue of (1) in shadow taxes t_i^*)

$\Longrightarrow$ $$\frac{\sum_i t_i^* s_{ki}}{x_k} = -(1-b) \qquad k = 1, \cdots, n$$

or, $$\frac{\sum_i t_i s_{ki}}{x_k} = -(1-b) + \frac{t_P s_{k1}}{x_k} \qquad k = 1, \cdots, n$$

- Goods more complementary with x_1 ($s_{k1} < 0$) discouraged more
- For x_1, $s_{11} < 0$, so unambiguously discouraged

Heterogeneous Households: The Social Welfare Function

Assumed properties of Social Welfare Function (SWF)

1. Welfarism: SWF depends only on utilities (consequentialism): $W(n_1, u_1, \cdots, n_h, u_h)$
2. Pareto principle: $W(\cdot)$ increasing in u_i
3. Symmetry, anonymity: u_i, u_j enter $W(\cdot)$ in same way
4. Quasi-concavity: convex to the origin social indifference curves

Simplest SWF satisfying these properties:

$$W(n_1, u_1, \cdots, n_h, u_h) = \sum_{i=1,h} n_i \frac{(u^i)^{1-\rho}}{1-\rho} = \sum n_i w(u_i), \; 0 \leq \rho \leq \infty$$

where $\rho = -w'' u / w'$: coefficient of aversion to inequality

$\rho = 0$: utilitarian; $\rho = \infty$: maximin

Note equivalence of concavity of $w(\cdot)$ and $u(\cdot)$

First-Best Utility Possibilities Frontier (UPF)

To characterize optimal redistribution between two household-types

Three cases

- Fixed labor supply, identical utility functions
- Fixed labor supply, different utility functions
- Variable labor supply, identical utility functions

Fixed labour supply, identical preferences

- n_1, n_2 type-1's and -2's with incomes y_1, y_2
- Lump-sum taxes T_1, T_2
- Identical utilities $u(c) = u(y - T)$, $u' > 0 > u''$
- Government budget $n_1 T_1 + n_2 T_2 = 0$

Effect of tax changes: $du^1 = -u_c^1 dT_1, \; du^2 = -u_c^2 dT_2$

Slope of UPF: $$\frac{du^2}{du^1} = \frac{u_c^2}{u_c^1}\frac{dT_2}{dT_1} = -\frac{n_1}{n_2}\frac{u_c^2}{u_c^1} \quad \text{(symmetric)}$$

Social welfare maximized at $u^1 = u^2$

Fixed Labor Supply, Different Utility Functions

Assume type-2's are better 'utility generators':

$$u^2(c) > u^1(c), \quad u_c^2(c) > u_c^1(c)$$

Slope of UPF: $$\frac{du^2(c_2)}{du^1(c_1)} = -\frac{n_1}{n_2}\frac{u_c^2(c_2)}{u_c^1(c_1)}$$

At $c_1 = c_2 = c_e$: $u^2(c_e) > u^1(c_e)$, $u_c^2(c_e) > u_c^1(c_e)$, and

$$\left|\frac{du^2(c_e)}{du^1(c_e)}\right| = \frac{n_1}{n_2}\frac{u_c^2(c_e)}{u_c^1(c_e)} > \frac{n_1}{n_2} \implies \text{FIGURE 1}$$

1. *Utilitarian outcome (u):* $u_c^1(c_1) = u_c^2(c_2) \Rightarrow c_2 > c_1$ (give more consumption to the better utility generator)
2. *Maximin outcome (m):* $u^1(c_1) = u^2(c_2) \Rightarrow c_1 > c_2$ (give more consumption to the less efficient utility generator)
3. As aversion to inequality increases, move from Utilitarian to Maximin outcome

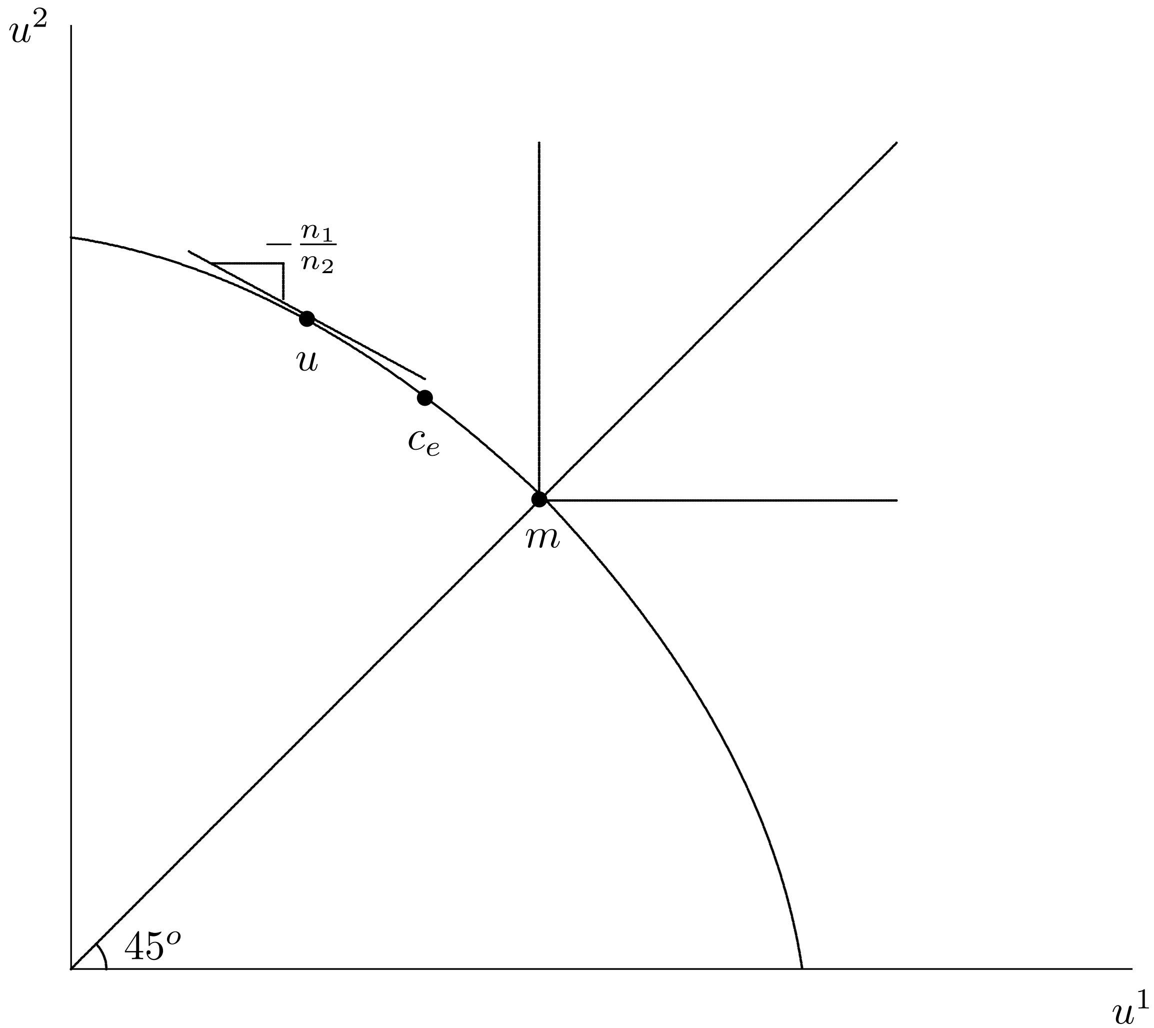

Figure 1 UPF with Different Utility Functions

Variable Labor Supply, Same Utility Functions

This is classical optimal income tax setting (Mirrlees)

- Common utility function: $u(c, \ell)$ (strictly concave)
- Wage rates differ: $w_2 > w_1$
- Linear production (& no government spending):
  $n_1 w_1 \ell_1 + n_2 w_2 \ell_2 = n_1 c_1 + n_2 c_2$
- Lump-sum taxes T_1, T_2

Representative household behavior

- Max $u(c, \ell)$ s.t. $c = w\ell - T$
- FOC: $-u_\ell / u_c = w \ \Rightarrow \ell(w, T)$
- Indirect utility: $v(w, T)$ with $v_w = \ell u_c > 0, \ v_T = -u_c < 0$
- Two types: $v^1(w_1, T_1), \ v^2(w_2, T_2)$

Properties of First-Best UPF

Move along First-Best UPF using lump-sum taxes T_1, T_2:

Slope: $$\frac{dv^2}{dv^1} = \frac{v_T^2 dT_2}{v_T^1 dT_1} = -\frac{n_1}{n_2}\frac{v_T^2}{v_T^1} = -\frac{n_1}{n_2}\frac{u_c^2}{u_c^1}$$

$\Rightarrow$ Concave to the origin since $v_{TT}^i < 0$

Points on UPF: FIGURE 2

- *Laissez Faire* (ℓ): $T_1 = T_2 = 0 \Rightarrow v^2 > v^1$
- *Maximin* (m): $v^1 = v^2 \Rightarrow$ On 45^o line
- *Utilitarian* (u): T_1, T_2 chosen such that $u_c^1 = u_c^2$ and $-u_\ell^i / u_c^i = w_i \Rightarrow v^1 > v^2$ if c normal (Mirrlees)

Intuition for utilitarian case: Efficient for high-wage person to supply more labor, while marginal utility of consumption equalized

Note: Tax may not be progressive even under Maximin (Sadka)

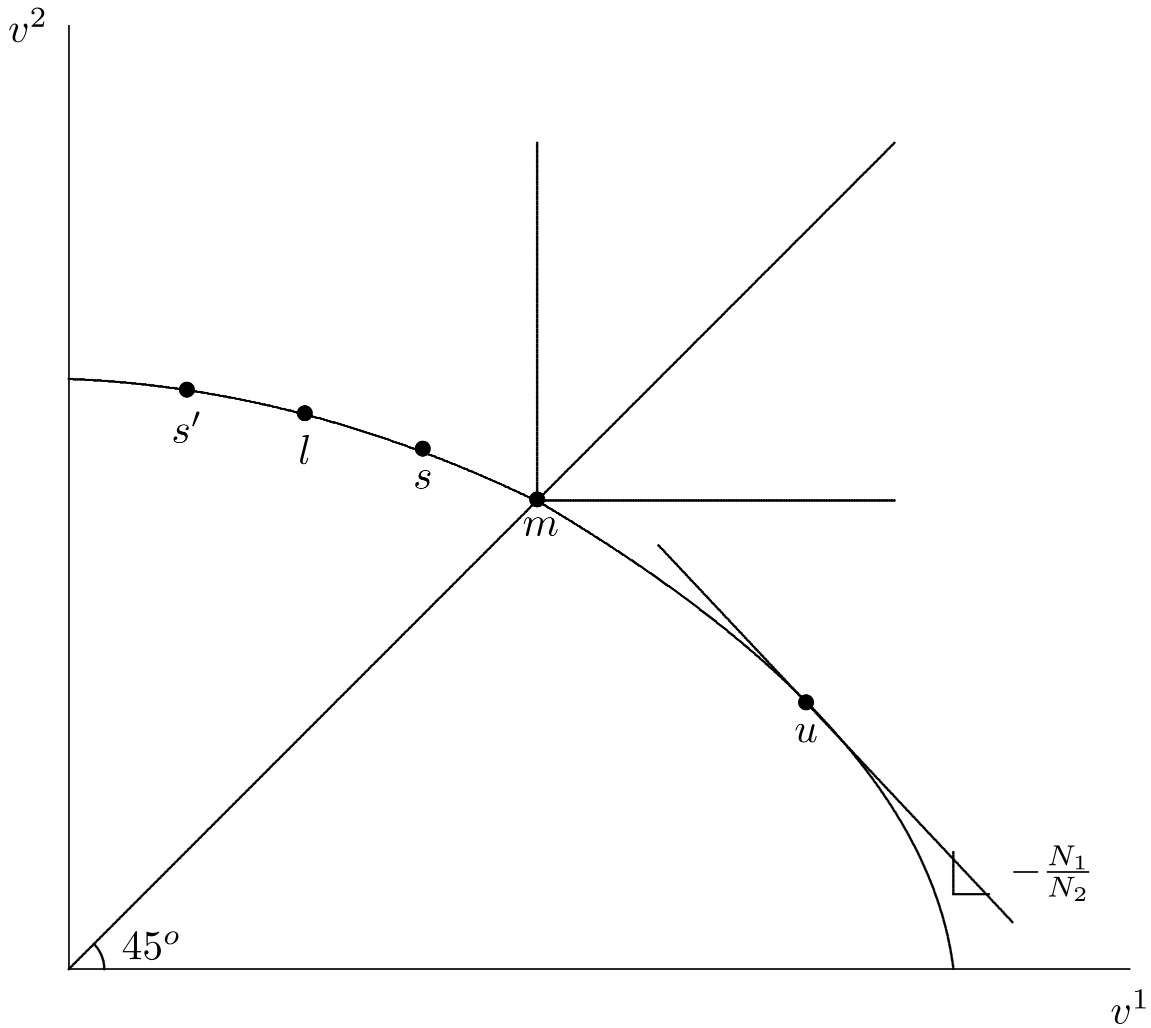

Figure 2 First-Best UPF

An Aside: Different Wages, Different Preferences

Suppose utility function is $u(c) - \phi_i h(\ell)$ with $c = w_i\ell - T_i$
(where T_i is based on wage rate)

Household characteristics

- Abilities: $w_2 > w_1$
- Preference for leisure: $\phi_2 > \phi_1$

Normative principles

- *Principle of Compensation:* Compensate for differences in ability: Persons with different wage and same preferences should achieve same utility (maximin)
- *Principle of Responsibility:* Do not compensate for differences in preference: Persons with same wage and different preferences should pay same tax

Result: Impossible to satisfy Principle of Compensation and Principle of Responsibility simultaneously $\Longrightarrow$ FIGURE 3

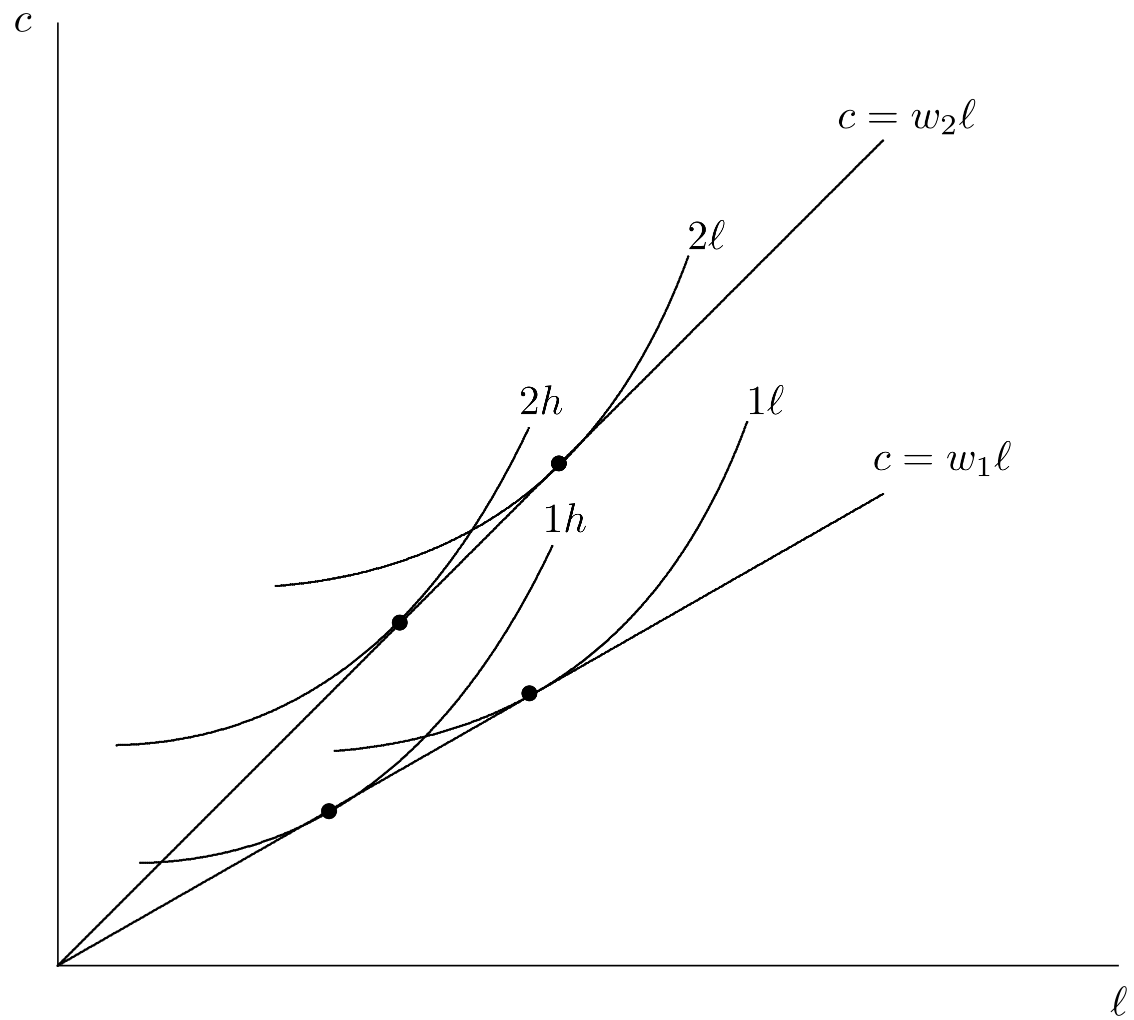

Figure 3 Principles of Compensation and Responsibility

Optimal Linear Taxation: Basic Results

- Suppose households vary in wage rates w_j, $j = 1, \cdots, h$, but have common preferences
- Government levies commodity taxes and equal per person lump-sum subsidy a
- Household indirect utilities $v^j(q, w_j, a)$
- Government objective is additive social welfare function $W(v^1) + W(v^2), \cdots, +W(v^j), \cdots, +W(v^h)$
- Deaton (1979): Uniform commodity taxes optimal if utility weakly separable and goods have linear Engel curves with identical slopes across households
- Satisfied by Gorman polar form utility function, whose expenditure function for person i takes the form $e^i(\mathbf{q}, u^i) = g^i(\mathbf{q}) + u^i f(\mathbf{q})$, where $f(\mathbf{q})$ is the same for all
- $\Longrightarrow$ Linear progressive income tax

Optimal Linear Taxation: Special Case

The setting: Household with wage rate w

- Assume two goods, x_1, x_2, and labor, ℓ
- Utility: $u(x_1, x_2, \ell)$
- Linear income tax t, a and commodity tax θ on x_2
- Producer prices for goods = unity
- Consumer budget $x_1 + (1+\theta)x_2 = (1-t)w\ell + a$

Household Lagrangian

$$\mathcal{L} = u(\cdot) - \alpha\big(x_1 + (1+\theta)x_2 - (1-t)w\ell - a\big) \quad \Longrightarrow$$

Solution:

$$x_1(\theta, t, a),\ x_2(\theta, t, a),\ \ell(\theta, t, a)$$

Indirect utility: $v(\theta, t, a)$

where envelope theorem implies

$$v_\theta = -\alpha x_2,\ v_t = -\alpha w\ell,\ v_a = \alpha$$

Optimal Linear Progressive Income Tax

Assume additive social welfare function:

$$\int_{\underline{w}}^{\overline{w}} W\big(v(\theta, t, a)\big)\,dF(w)$$

Government problem: Choose t, a given θ (assuming $R = 0$)

$$\mathcal{L} = \int_{\underline{w}}^{\overline{w}} \Big(W\big(v(\theta, t, a)\big) + \lambda\big(\theta x_2(\theta, t, a) + tw\ell(\theta, t, a) - a\big)\Big)\,dF(w)$$

First-order conditions evaluated at $\theta = 0$:

$$a: \qquad \int_{\underline{w}}^{\overline{w}} \left(W' v_a + \lambda\left(tw\frac{\partial \ell}{\partial a} - 1\right)\right) dF(w) = 0$$

$$t: \qquad \int_{\underline{w}}^{\overline{w}} \left(W' v_t + \lambda\left(w\ell + tw\frac{\partial \ell}{\partial t}\right)\right) dF(w) = 0$$

Some Definitions

Define:

$\beta(w) \equiv W' v_a = W' \alpha$ (marginal social utility of income)

$b(w) \equiv \beta(w)/\lambda + tw\partial\ell/\partial a$ (social value of government transfer)

Slutsky equation, where $w_n = (1-t)w$ and $s_{\ell\ell}$ is the own substitution effect:

$$\frac{\partial \ell}{\partial w_n} = s_{\ell\ell} + \ell \frac{\partial \ell}{\partial a} \quad \Longrightarrow$$

$$\frac{\partial \ell}{\partial t} = \frac{\partial \ell}{\partial w_n}\frac{\partial w_n}{\partial t} = -ws_{\ell\ell} - w\ell\frac{\partial \ell}{\partial a}$$

And, define the compensated elasticity of labor supply:

$$\epsilon_{\ell\ell} \equiv s_{\ell\ell}\frac{(1-t)w}{\ell} = \frac{tw}{\ell}s_{\ell\ell}\frac{1-t}{t}$$

Interpretation of First-Order Conditions

Using these definitions and $v_t = -\alpha w \ell$, the FOCs become:

$$a: \qquad \int \big(b(w) - 1\big) dF(w) = 0, \qquad \implies \qquad \overline{b} = 1$$

$$t: \qquad \int w\ell \Big(b(w) - 1 + \frac{t}{1-t}\epsilon_{\ell\ell}\Big) dF(w) = 0$$

So, using $y = w\ell$ and $E[b] = 1$,

$$\frac{t}{1-t} = -\frac{\int \big(b(w) - 1\big) w\ell dF(w)}{\int w\ell\epsilon_{\ell\ell} dF(w)} = -\frac{E[by] - E[b]E[y]}{\int w\ell\epsilon_{\ell\ell} dF(w)}$$

$$= -\frac{\text{Cov}[b, y]}{\int y\epsilon_{\ell\ell} dF(w)} = -\frac{\text{Equity } (-)}{\text{Efficiency } (+)}$$

Differential Commodity Taxes

Let $\mathcal{W}(\theta)$ = value function from optimal linear income tax problem. Using the envelope theorem and $v_\theta = -\alpha x_2$:

$$\left.\frac{d\mathcal{W}}{d\theta}\right|_{\theta=0} = \int_{\underline{w}}^{\overline{w}} \left(-\beta x_2 + \lambda\Big(x_2 + tw\frac{\partial \ell}{\partial \theta}\Big)\right) dF(w)$$

The Slutsky equation corresponding to $\partial \ell / \partial \theta$ is:

$$\frac{\partial \ell}{\partial \theta} = s_{\ell 2} - x_2 \frac{\partial \ell}{\partial a}$$

Substituting this into $d\mathcal{W}/d\theta$ and rearranging, we obtain:

$$\frac{1}{\lambda}\left.\frac{d\mathcal{W}}{d\theta}\right|_{\theta=0} = \int_{\underline{w}}^{\overline{w}} x_2 \left(-\frac{\beta}{\lambda} + 1 + tw\frac{s_{\ell 2}}{x_2} - tw\frac{\partial \ell}{\partial a}\right) dF(w)$$

The FOC on t above can be written:

$$\int_{\underline{w}}^{\overline{w}} y \left(-\frac{\beta}{\lambda} + 1 + tw\frac{s_{\ell\ell}}{\ell} - tw\frac{\partial \ell}{\partial a}\right) dF(w) = 0$$

Differential Commodity Taxes, cont'd

Deaton shows that optimal $\theta = 0$ if a) preferences are weakly separable in goods, and b) Engel curves are linear.

Special case: Quasilinear preferences
$u(x_1, x_2, \ell) = x_1 + b(x_2) - h(\ell)$

Then, $\ell = \ell\big((1-t)w\big),\ x_2 = x_2(1+\theta)$
$\Longrightarrow$ Deaton conditions satisfied

Since $\dfrac{\partial \ell}{\partial \theta} = \dfrac{\partial \ell}{\partial a} = 0$ and x_2 independent of w

FOC on a becomes :

$$\int_{\underline{w}}^{\overline{w}} \big(\beta - \lambda\big) dF(w) = 0$$

and,

$$\left.\frac{d\mathcal{W}}{d\theta}\right|_{\theta=0} = \int_{\underline{w}}^{\overline{w}} \big(-\beta x_2 + \lambda x_2\big) dF(w) = 0$$

$\Longrightarrow \theta = 0$ is optimal

Production Efficiency: Diamond and Mirrlees

- Let $v^j(\mathbf{q}) = \max u(\mathbf{x}^j)$ s.t. $\sum_{i=0}^{n} q_i x_i^j = 0$ be j's utility
- Let $\mathbf{y}^k \in \mathbf{Y}^k$ be producer k's vector of production, where $\mathbf{Y}^k$ is k's feasible production set
- The government maximizes some social welfare function $W(v^1(\mathbf{q}), \cdots, v^h(\mathbf{q}))$ subject to $\sum_k \mathbf{x}^j + \mathbf{g} = \sum \mathbf{y}^k \in \sum_k \mathbf{Y}^k$ where $\mathbf{g}$ is net government production

Suppose $\sum \mathbf{y}^k$ lies in the interior of $\sum \mathbf{Y}^k$. The government can choose $\mathbf{q}$ independently of $\mathbf{p}$. If it reduces q_i for some good that has a positive net demand by consumers, social welfare will rise, and the increase of production is feasible. Thus, in an optimum, $\sum \mathbf{y}^k$ must be on the boundary of $\sum \mathbf{Y}^k$.

Intuition: With production inefficiency, reduction in consumer price of a good consumed increases utility; increased demand can be satisfied without sacrificing other goods

Taxation of profits important, though violation has unclear effects

Implications and Caveats of Production Efficiency Theorem

- Do not tax producer inputs (case for VAT)
- Use producer prices for public production (CBA rule)
- Caution: only applies if taxes are optimal, though implications of non-optimal taxes not obvious
- Newbery (1986): If only subset of commodities' consumption can be taxed optimally, welfare-improving to impose small tax on production of either untaxed or taxed commodity
- Choice of VAT versus trade taxes in LDCs with large informal sector: VAT preserves production efficiency in formal sector; trade taxes indirectly tax pure profits
- If skilled and unskilled labor imperfect substitutes, public sector can affect relative wages by increasing demand for unskilled labor inducing production inefficiency (Naito)
- Production Efficiency violated internationally (Keen-Wildasin)

Production Efficiency Theorem still a useful benchmark